Allocation of Quantum resources in Optical networks

R. Salazar, J. Czartowski, I. Vagniluca, D. Ribezzo, A. Zavatta, N. Farrugia, J. A. Briffa, A. Xuereb

Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, 30-348 Kraków, Poland
CNR - Istituto Nazionale di Ottica (CNR-INO) Largo E. Fermi 6, 50125 Firenze, Italy
Department of Physics "Ettore Pancini" University of Naples "Federico II", 80126 Naples, Italy
LENS and Dipartimento di Fisica e Astronomia Università di Firenze, Italy
University of Malta, Msida MSD2080, Malta

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Reliability of Quantum resources in Quantum networks

Quantum networks and most prominently the Quantum Internet, are one of the research fields of quantum technologies that promise near-future applications. Furthermore, there exist a hierarchy of development stages that depends on the resources implemented and functionality for a quantum network [1]. The early stages of this hierarchy are likely to be the first to be implemented in real communication networks because of their minimum requirement of Quantum resources. Additionally, the above networks will be relevant in the future scenario as proxies for those who do not directly access more developed networks due to a lack of quantum resources.

The second stage of a Quantum Internet considers only prepare-and-measure quantum devices [1], which is the first to offer end-to-end quantum functionality. For example, it enables end-to-end QKD without the need to trust intermediary repeater nodes. Informally, this stage allows any node to prepare a one-qudit state and transmit the resulting state to any other node, which then measures it.

Like in other quantum technologies, a crucial problem for quantum networks is noise or failure in devices. However, at the second stage of the previous hierarchy, advanced methods such as quantum error correction are unavailable, and more involved methods are needed. Here, we propose an experimental realization for a method that preserves the quantum advantage of a prepare-and-measure communication protocol based on an appropriate choice of the quantum resource necessary for the protocol.

In classical networks, reliability means that the average performance of the communication between two particular nodes under the possible failure of the system is above the desired threshold [2, 3]. For quantum networks, the threshold is identified with a classical bound or, more generally, with the bound of devices without a specific quantum resource crucial for the task of interest [4].

A strategy to improve the network's reliability is to introduce a redundant allocation of the resources necessary for the quantum communication advantage between the target nodes. In quantum networks, such allocation depends on the multipartite properties of the resource and to determine its optimality requires a more detailed analysis [4]. However, there exists an optimal solution for the simplest multipartite network that exploits the incompatibility of quantum measurements for the communication between nodes [4].

The simplest multipartite communication network consist in two nodes connected by a physical system with two degrees of freedom s_1 and s_2 as shown in Figure 1.

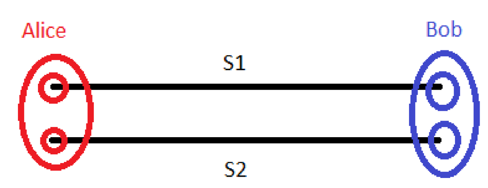

Figure 1:
Simplest prepare-and-measure network with redundant allocation of the quantum resource.

Following the standard nomenclature, we will call the encoding node Alice and Bob the decoding node. If degrees of freedom s_1, s_2 have prior probabilities $1-p(s_1)$ and $1-p(s_2)$ of failure, we have three possible nonzero communication scenarios between Alice and Bob: i) communication using both s_1, s_2, ii) only using s_2 or iii) only using s_1. For a pictorical visualization of each scenario see Figure 2.

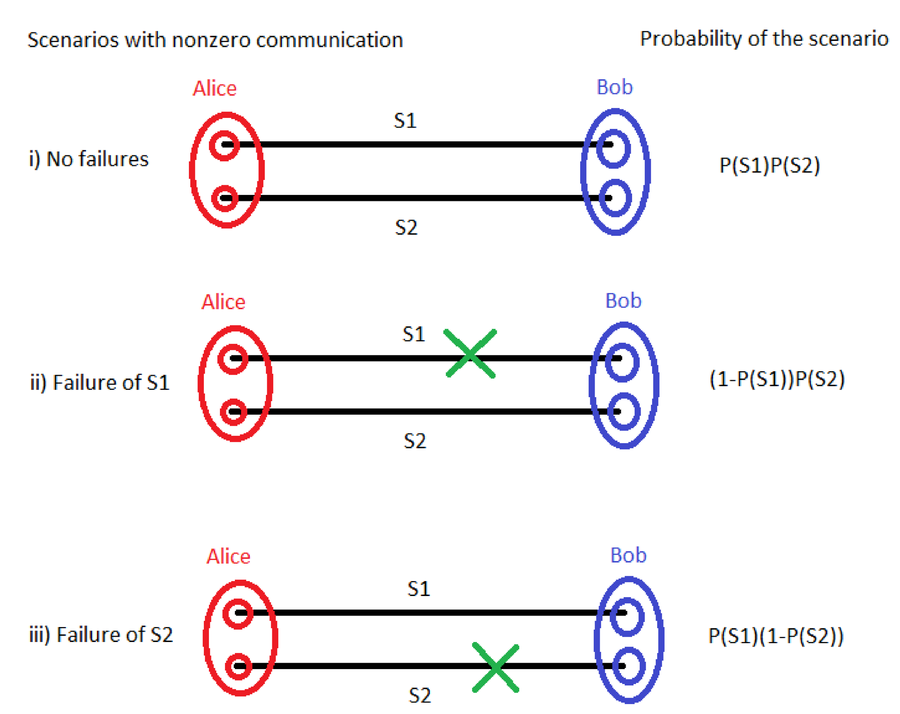

Figure 2:
The three possible communication scenarios: i) Non failure of either degree of freedom with probability $\pi_{s_1s_2} = p(s_1)\,p(s_2)$, ii) Failure of degree s_1 alone with probability $\pi_{s_2} = (1-p(s_1))\,p(s_2)$ and $\pi_{s_1s_2} = p(s_1)\,p(s_2)$, iii) Failure of only degree s_2 with probability $\pi_{s_1} = p(s_1)(1-p(s_2))$

In the general scenario the probability of success P_{succ} of the communication protocol depends in general of the bipartite devices of both Alice and Bob. However, in this work we will focus on the dependence of a particular protocol to the resourcefulness of the bipartite measurement $\mathbf{M}_{s_1s_2}$ used by Bob to decode. To facilitate the analysis we define the terms A_α for the communication advantage in each scenario α of nonzero communication:

$$\begin{aligned} A_{12} &= [[P_{succ}(\mathbf{M}_{s_1s_2}) - P_{cl}]] && (1)\\ A_1 &= [[P_{succ}(\mathbf{M}_{s_1}) - P_{cl}]] && (2)\\ A_2 &= [[P_{succ}(\mathbf{M}_{s_2}) - P_{cl}]] && (3)\end{aligned}$$

with the terms in brackets $[[x]]$ equal to x if $x>0$ and 0 otherwise. We call the list $\mathcal{A} = \{A_{12}, A_1, A_2\}$ a *resource allocation* [4]. The performance Φ of the device $\mathbf{M}_{s_1s_2}$ evaluates the average of communication advantage provided by $\mathbf{M}_{s_1s_2}$ over the classical treshold P_{cl}, naturally described in terms of the resource allocation:

$$\Phi(\mathbf{M}_{s_1s_2}) = \pi_{s_1s_2}A_{12}(\mathbf{M}_{s_1s_2}) + \pi_{s_1}A_1(\mathbf{M}_{s_1}) + \pi_{s_2}A_2(\mathbf{M}_{s_2}) \qquad (4)$$

The performance $\Phi(\mathbf{M}_{s_1s_2})$ will be our figure of merit and the object of study in the experimental test. Precisely, we investigate how the elements of resource allocation $\mathcal{A}$ change as the optimal device $\mathbf{M}^{opt}_{s_1s_2}$ is degraded by a depolarizing noise. Such test will determine experimentally the robustness of the network's communication advantage under noise and suggest strategies for preserving the optimal allocation of the resource.

Another important problem in communication networks is bandwith allocation [15]. We would like to remark that the experimental test proposed could also be applied to study the analogue problem of a fair distribution of the quantum advantage, between users which control different degrees of freedom. Indeed, reference [4] also provides the performance Φ' associated with a proportional fair distribution:

$$\Phi'(\mathbf{M}_{s_1s_2}) = \log[A_{12}(\mathbf{M}_{s_1s_2})] + \log[A_1(\mathbf{M}_{s_1})] + \log[A_2(\mathbf{M}_{s_2})] \qquad (5)$$

From the above follows that an experimental test of the resource allocation to study the reliability Φ allows the study of proportional fairness Φ'.

Communication protocol with optimal allocation of the quantum resource

Quantum random access codes (QRACS) is a prepare-and-measure communication protocol with quantum advantage which has been extensively studied in quantum information [5, 6, 7, 8, 9, 10, 11, 12], including experimental applications [13, 14]. In the (n,d)-QRAC scheme Alice receives n dits, $\mathbf{x} = (x_1, ..., x_n)$. She can send one qudit to Bob. In addition, Bob receives a number $j \in \{1, ..., n\}$ and his task is to guess the corresponding dit x_j. He does this by performing a measurement, depending on j, thereby obtaining an outcome z. The enconding-decoding is successful if $z = x_j$. The total success probability is usually taken to be either the worst case success probability or the average success probability, and when calculating these numbers it is assumed that the inputs for both Alice and Bob are uniformly distributed.

The strategy of Alice and Bob consist of d^n quantum states for encoding and n d-outcome measurements for decoding, all defined for a d-level quantum system. We denote by $\mathcal{E}$ the encoding map and $M_1, ..., M_n$ the measurements. Hence, $\mathcal{E}(\mathbf{x})$ is a quantum state for each $\mathbf{x}$, and $M_1, ..., M_n$ are d-outcome positive operator valued measures (POVMs). The average success probability is then given as:

$$P\left(\mathcal{E}, \{M_k\}_{k=1}^{n}\right) = \frac{1}{nd^n}\sum_{\mathbf{x}} \mathrm{Tr}\left[\sum_{k=1}^{n}\mathcal{E}(\mathbf{x})M_k(x_k)\right] \qquad (6)$$

We denote by $P^{n,d}_{qrac}$ the best achievable average success probability in (n,d)-QRAC. In the case $n=2$, it is known from that [5]:

$$P^{2,d}_{qrac} = \frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right) \qquad (7)$$

which is satisfied by a mutually unbiased pair of measurements in every dimension d [5, 7, 9, 11, 12]. The relevance of QRACs lies in their advantage over any classical random access code (RAC), in which Alice is allowed to send to Bob a single dit rather than a qudit and Bob only has access to classical measurements. For $(2,d)$-RACs, the optimal average success probability is [5]:

$$P^{2,d}_{rac} = \frac{1}{2}\left(1+\frac{1}{d}\right) \qquad (8)$$

For each set of decoding measurements $\mathbf{M} = \{M_k\}_{k=1}^{n}$ the optimal quantum strategy is achieved when $\mathcal{E}(\mathbf{x})$ is an eigenvalue of the operator $\sum_{k=1}^{n} M_k(x_k)$ [5, 12]. In consequence we can write the maximal success probability $P_{qrac}(\mathbf{M})$ using $\mathbf{M}$ as:

$$P_{qrac}(\mathbf{M}) = \frac{1}{nd^n}\sum_{\mathbf{x}}\left\|\sum_{k=1}^{n} M_k(x_k)\right\| \qquad (9)$$

where $\|\cdot\|$ is the operator norm. The above lead to a simple measure $\mathcal{M}(\mathbf{M})$ of the advantage provided by $\mathbf{M}$ over any classical strategy in $n=2$:

$$\mathcal{M}(\mathbf{M}) = \max\left\{P_{qrac}(\mathbf{M}) - \frac{1}{2}\left(1+\frac{1}{d}\right), 0\right\} \qquad (10)$$

In [12] it was shown incompatibility of $\mathbf{M} = \{M_1, M_2\}$ as a necessary condition for an advantage $\mathcal{M}(\mathbf{M}) > 0$ in $(2,d)$-QRACs, with maximum advantage when $\mathbf{M}$ is a Mutually unbiased (MU) pair, i.e. two sets of projectors $\{|\varphi_x\rangle\langle\varphi_x|\}_{x=1}^{d}, \{|\psi_y\rangle\langle\psi_y|\}_{y=1}^{d}$ with $\{|\varphi_x\rangle\}_{x=1}^{d}$ and $\{|\psi_y\rangle\}_{y=1}^{d}$ two orthonormal bases which satisfy:

$$|\langle\psi_y \mid \varphi_x\rangle|^2 = \frac{1}{d} \quad \forall x, y \in \{1, ..., d\} \qquad (11)$$

Is straight forward to demonstate that Theorem 1 in reference [4] holds for the monotone $\mathcal{M}(\mathbf{M}_{s_1s_2})$ from (10) as it satisfies all the properties used in the proof. The content of Theorem 1 in [4] is that for an appropriate monotone like $\mathcal{M}(\mathbf{M}_{s_1s_2})$ the allocation $\mathcal{A} = \{A_{12}, A_1, A_2\}$ which optimizes the performance $\Phi(\mathbf{M}_{s_1s_2})$ for degrees of freedom of any dimension d is given by a separable MU pair $\mathbf{M}^{opt}_{s_1s_2}$:

$$\mathbf{M}^{opt}_{s_1s_2} = \left\{\left\{|\varphi_{x_1}\rangle_{s_1}\langle\varphi_{x_1}|\otimes|\varphi_{x_2}\rangle_{s_2}\langle\varphi_{x_2}|\right\}_{x_1,x_2=1}^{d}, \left\{|\psi_{y_1}\rangle_{s_1}\langle\psi_{y_1}|\otimes|\psi_{y_2}\rangle_{s_2}\langle\psi_{y_2}|\right\}_{y_1,y_2=1}^{d}\right\} \qquad (12)$$

with $\{|\varphi_x\rangle\}_{x=1}^{d}$ and $\{|\psi_y\rangle\}_{y=1}^{d}$ two orthonormal bases satisfying the MU contition (11). In consequence, the optimal allocation $\mathcal{A}^{opt} = \{A^{opt}_{12}, A^{opt}_1, A^{opt}_2\}$ is:

$$\begin{aligned} A^{opt}_{12} &= \frac{1}{2}\left(\frac{1}{d}-\frac{1}{d^2}\right) && (13)\\ A^{opt}_{1} &= \frac{1}{2}\left(\frac{1}{\sqrt{d}}-\frac{1}{d}\right) && (14)\\ A^{opt}_{2} &= \frac{1}{2}\left(\frac{1}{\sqrt{d}}-\frac{1}{d}\right) && (15)\end{aligned}$$

The proposed experimental test is to implement a QRAC protocol in a simple two-nodes multipartite communication network and study the variations induced by noises ϵ, which could affect the optimal performance of the protocol

We model the noise as the application of a polarization transformation to $\mathbf{M}^{opt}_{s_1s_2}$, this is a measurement pair $\mathbf{M}_{s_1s_2}(\epsilon_1, \epsilon_2)$:

$$\begin{aligned} M_{\epsilon_1} &= \left\{(1-\epsilon_1)\,|\varphi_{x_1}\rangle_{s_1}\langle\varphi_{x_1}|\otimes|\varphi_{x_2}\rangle_{s_2}\langle\varphi_{x_2}| + \epsilon_1\frac{1}{d^2}I_{s_1}\otimes I_{s_2}\right\}_{x_1,x_2=1}^{d} && (16)\\ M_{\epsilon_2} &= \left\{(1-\epsilon_2)\,|\psi_{y_1}\rangle_{s_1}\langle\psi_{y_1}|\otimes|\psi_{y_2}\rangle_{s_2}\langle\psi_{y_2}| + \epsilon_2\frac{1}{d^2}I_{s_1}\otimes I_{s_2}\right\}_{y_1,y_2=1}^{d} \end{aligned}$$

The experiment considers the simplest case of $d=2$ and then study the variation of $\mathcal{A}^{opt}$ with respect to the noise parameters ϵ_1, ϵ_2. With this purpose we will measure the values of the resource allocation $\{A_{12}(\epsilon_1,\epsilon_2), A_1(\epsilon_1,\epsilon_2), A_2(\epsilon_1,\epsilon_2)\}$ for different noise values in measurements of the form (17) and compare them with those of $\mathcal{A}^{opt}$.

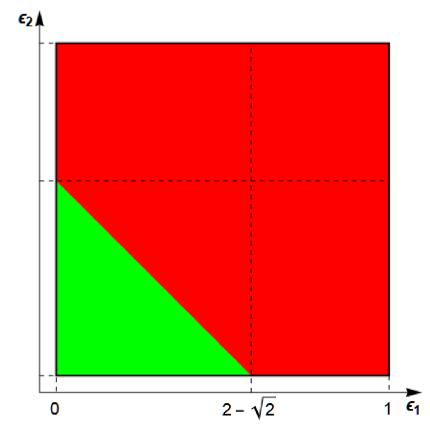

Figure 3: Green: Region of nonzero quantum advantage for single-qubit protocol, Red: region of no advantage

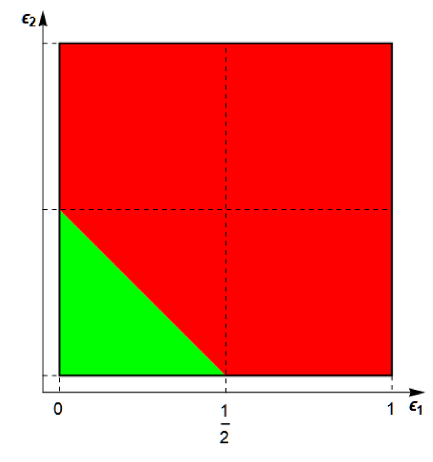

Figure 4: Green: Region of nonzero quantum advantage for two-qubit protocol, Red: region of no advantage

Experimental test of resource allocation

Figure 5 display a scheme presenting the pulses and the properties detected to form the degrees of freedom exploited by the resourceful measurements in the protocol. Figure 6 shows the set-up for the protocol exploiting the first degree of freedom: arrival time in path A and phase on path B. Conversely, the second degree of freedom considers the phase on path A and time of arrival on path B. Figure 7 exhibit the set-up that exploits both degrees of freedom, where detectors determine the time of arrival and phase along paths A and B.

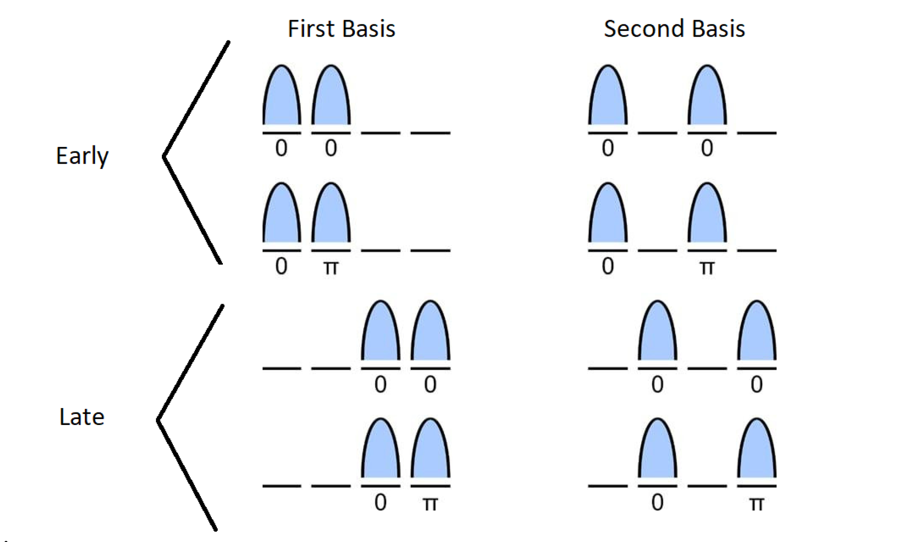

Figure 5:
Each of the columns represents one of the two MUBs used to construct the measurements of the protocols. The first MUB is associated with path A after the beamsplitter, while the second MUB correspond to path B. Detectors measure the arrival time distinguishing between early or late pulses, and the interferometers on each arm split the pulses in phase [00] or dephase [0π] ones.

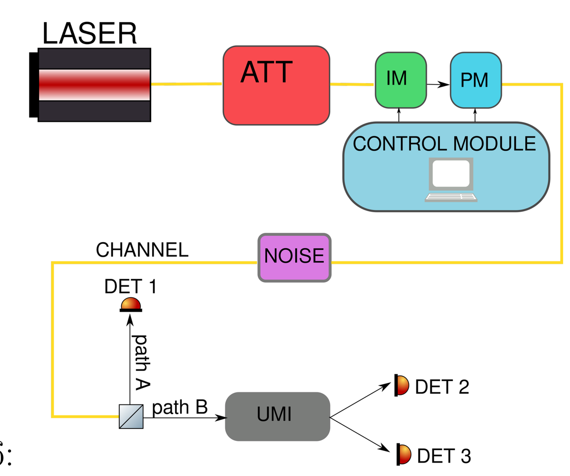

Figure 6:
Experimental set-up for a QRAC protocol encoding two messages in a single-qubit. The states are encoded in pulses created at the transmitter's side by an attenuated 1550 nm continuum laser passing through an intensity modulator (IM) and a phase modulator (PM) that provide the right time spacing and phase to the train of pulses; a programmable board controls the modulators. A source of variable noise is implemented in the channel. At the receiver's side, a beamsplitter makes it possible to project into path A, where detector one measures the arrival time (first MUB), or to path B, where an unbalanced Mach-Zehnder interferometer (UMI) separate the phases to measure pulses in phase [00] in detector two or dephased pulses [0π] in detector three (second MUB).

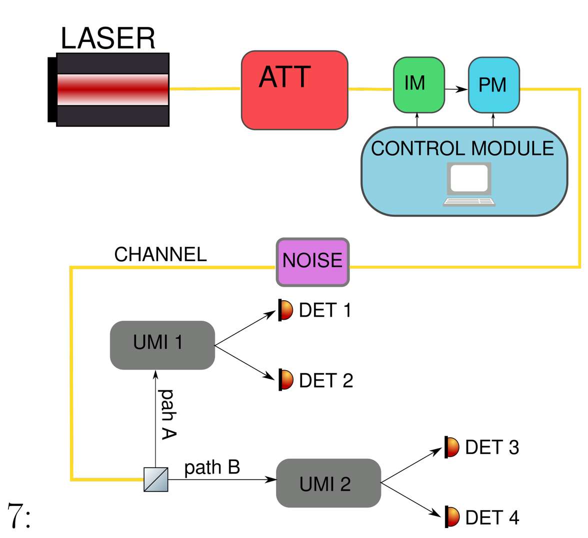

Figure 7:
The set-up of figure 6 is extended and adapted for performing a QRAC protocol encoding two messages in a two-qubit system. The receiver uses two unbalanced Mach-Zehnder interferometers, UMI 1 and UMI 2, where UMI 1, which reads the states in the first MUB, introduces half the time delay of UMI 2, which reads in the second MUB.

The implementation of the experimental set-up is being realized at the Quantum Communication Laboratories of the Istituto Nazionale di Ottica (INO) in Florence, Italy, by the Quantum Communication and Quantum Information Group.

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