Thermal recall: Memory-assisted Markovian thermal processes

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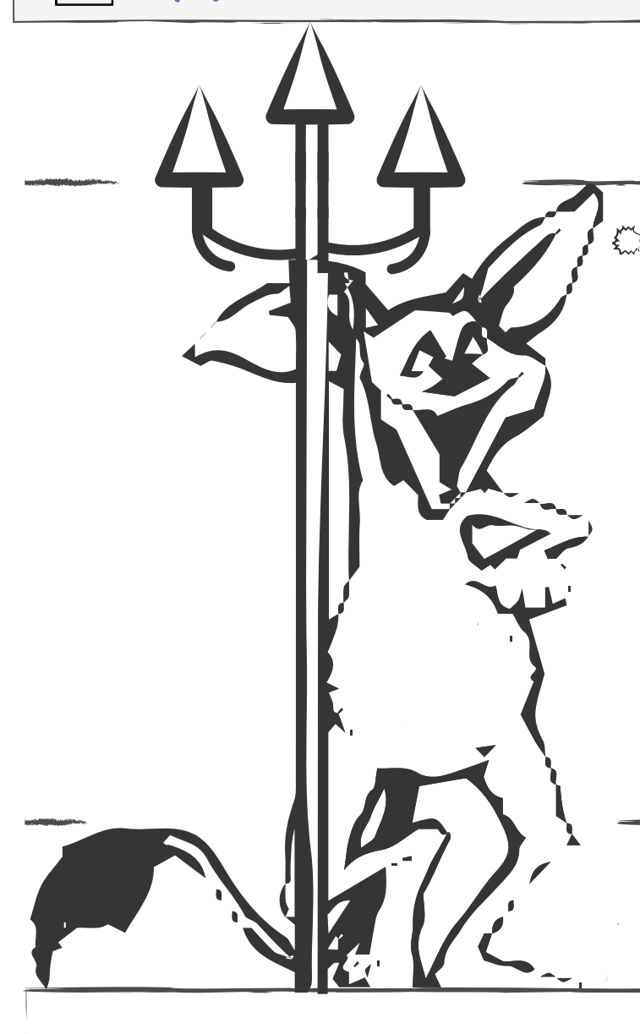
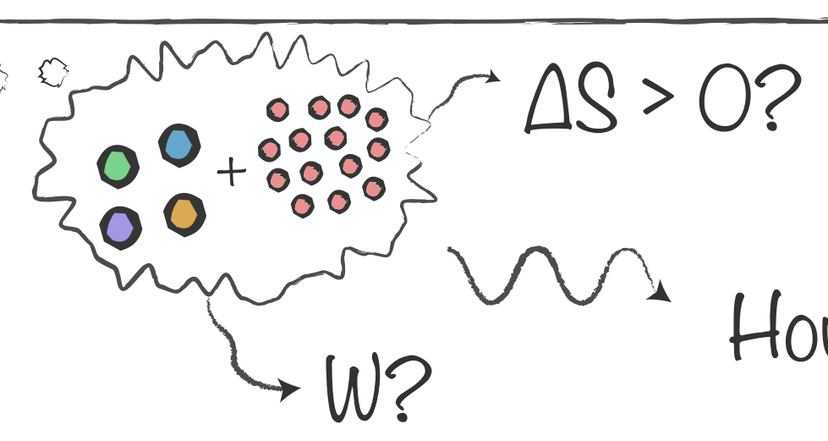




How to describe the thermodynamic of a small system?

- No memory → Markovian thermal process (MTP)
- **Finite-memory** → How to interpolate? → **Memory-assisted MTP (MeMTP)**
- (infinite) memory → Thermal operations (TO)

Setting the scene

i. Minimal assumptions on the joint system-bath dynamics: ● + ●●● — joint system closed — arbitrarily strong correlations

$$\mathcal{E}(\rho) = \mathrm{tr}_E\left[U\left(\rho\otimes\underbrace{\frac{e^{-\beta H_E}}{\mathrm{Tr}[e^{-\beta H_E}]}}_{\gamma}\right)U^\dagger\right] \quad \text{with} \quad [U, H\otimes \mathbb{1}_E + \mathbb{1}\otimes H_E] = 0$$

ii. Joint system-bath undergoes an open dynamics: ● + ●●● — weak coupling, large bath size, quickly decaying correlations...

$$\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \mathcal{L}_t[\rho(t)] \quad \text{with} \quad \mathcal{L}_t(\rho) = \sum_i r_i(t)\left[L_i(t)\rho L_i^\dagger(t) - \frac{1}{2}\left\{L_i^\dagger(t)L_i(t), \rho\right\}\right]$$

☐ Assumption: $(\rho, H) = \left(\sum p_i |e_i\rangle\langle e_i|, \sum E_i |e_i\rangle\langle e_i|\right)$ $\xrightarrow[\text{question:}]{\text{Fundamental}}$ Given **p**, **q**, and a thermal bath at β: ● + ●●● $\xrightarrow{?}$ ●

Energy-incoherent state → $\mathbf{p} = (p_1, ..., p_d)$

Answer in a nutshel:

Memoryless	$\mathbf{p} \xrightarrow{\text{MTP}} \mathbf{q}$	$\Longleftrightarrow$	$\mathbf{p} \gg_\beta \mathbf{q}$
Finite memory	$\mathbf{p} \xrightarrow{\text{MeMTP}} \mathbf{q}$	$\Longleftrightarrow$	$\mathbf{p}\otimes\gamma_M \gg_\beta \mathbf{q}\otimes\gamma_M$
Full control	$\mathbf{p} \xrightarrow{\text{TO}} \mathbf{q}$	$\Longleftrightarrow$	$\mathbf{p} \succ_\beta \mathbf{q}$

$\succ_\beta$: thermomajorisation

M. Horodecki & Jonathan Oppenheim, Nat.Commun (2013)

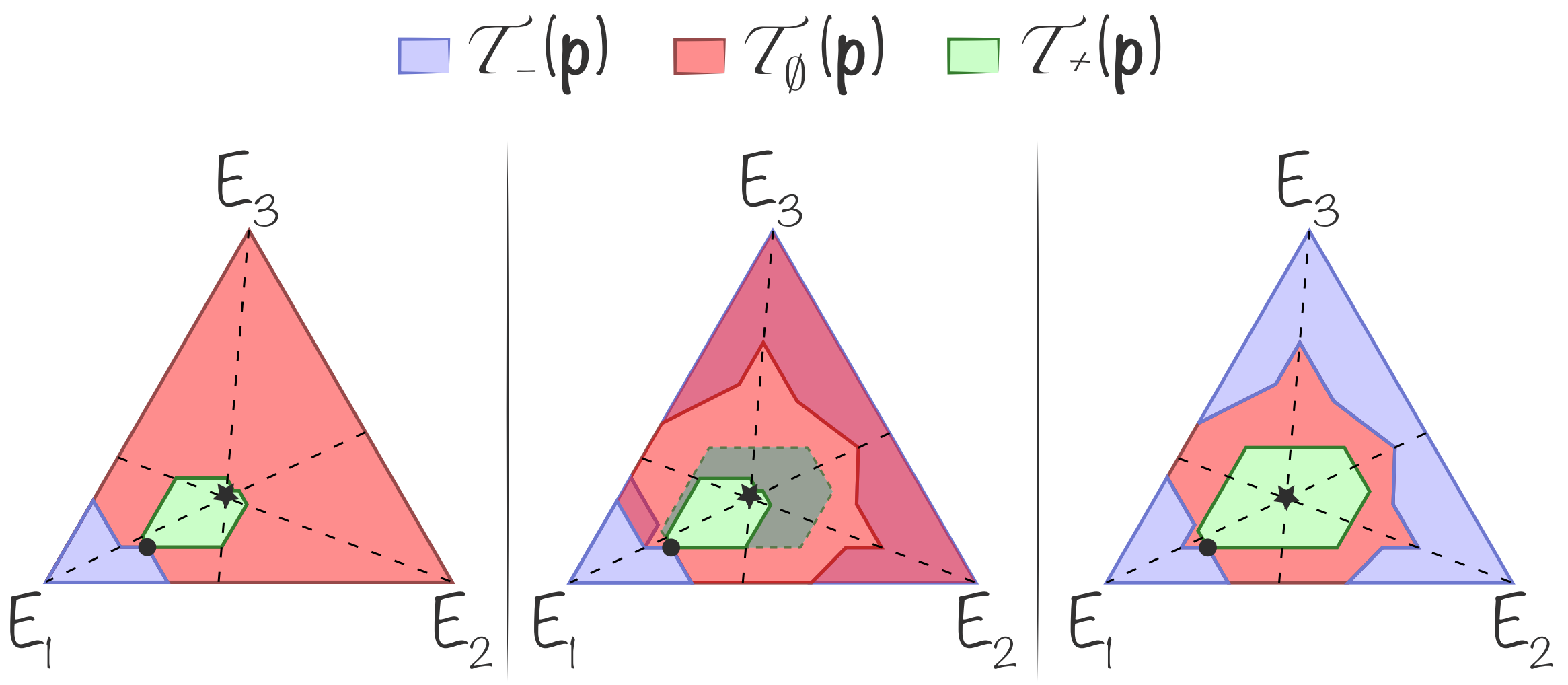
$\gg_\beta$: continuous thermomajorisation

M. Lostaglio & K. Korzekwa Phys. Rev. A, (2022)

A gap to be filled...

Thermal operations vs Markovian thermal processes

☐ $\mathbf{p} = (0.7, 0.2, 0.1)$, $E_s = (0, 1, 2)$ and $\beta = 0.3$



Some definitions/remarks

○ Total variation distance: $\delta(\mathbf{p},\mathbf{q}) := \frac{1}{2}\sum_{i=1}^{d}|p_i - q_i|$

○ If β = 0, then: $\Pi = \Pi_{i_m j_m} \dots \Pi_{i_1 j_1}$ with $\Pi_{ij} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{ij}$

permutation decomposability

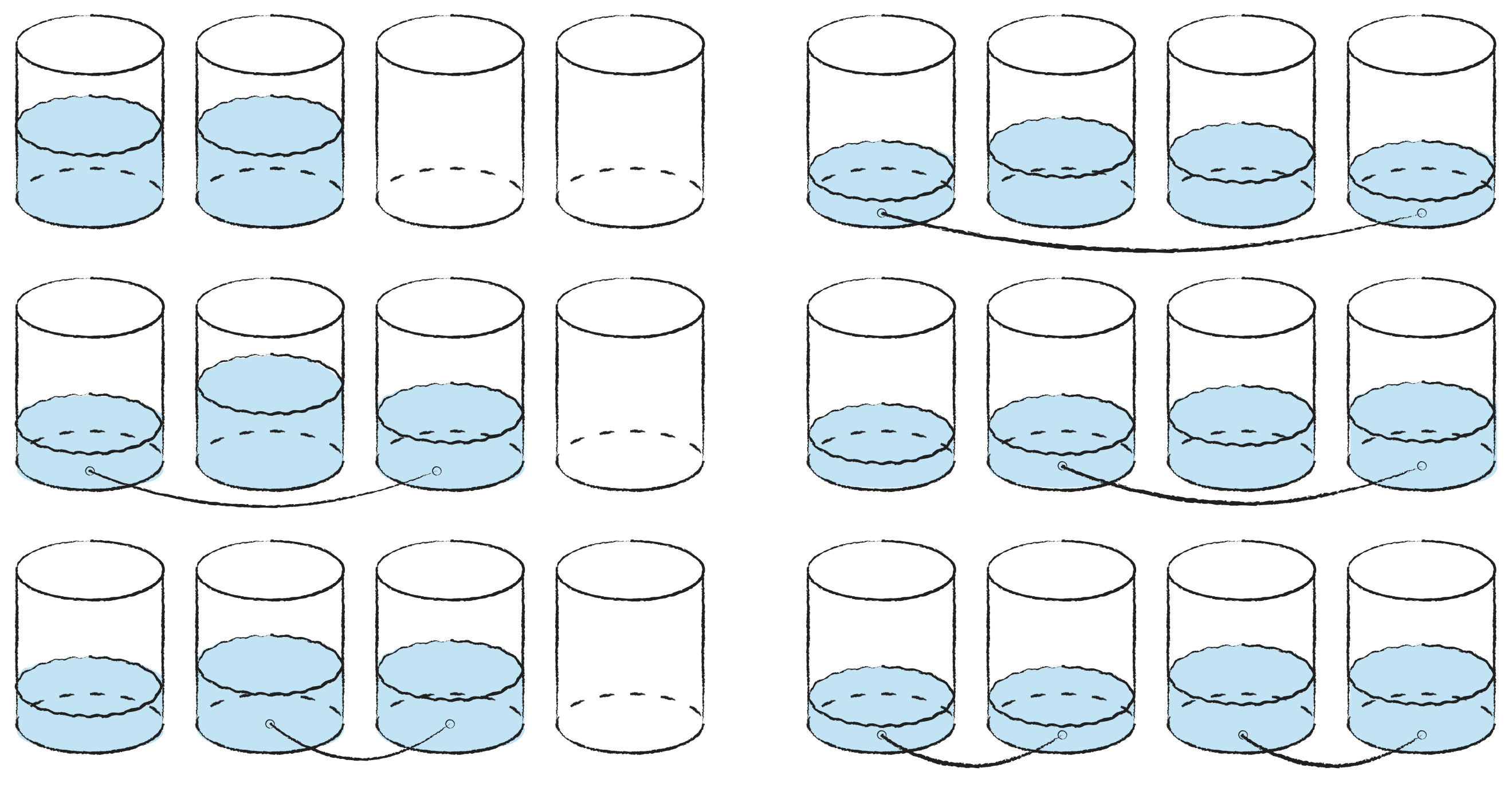
○ If β ≠ 0, then: $\Pi^\beta \neq \Pi^\beta_{i_m j_m} \dots \Pi^\beta_{i_1 j_1}$ with $\Pi^\beta_{ij} = \begin{pmatrix} 1 - e^{-\beta(E_j - E_i)} & 1 \\ e^{-\beta(E_j - E_i)} & 0 \end{pmatrix}_{ij}$

β-permutation (non) decomposability

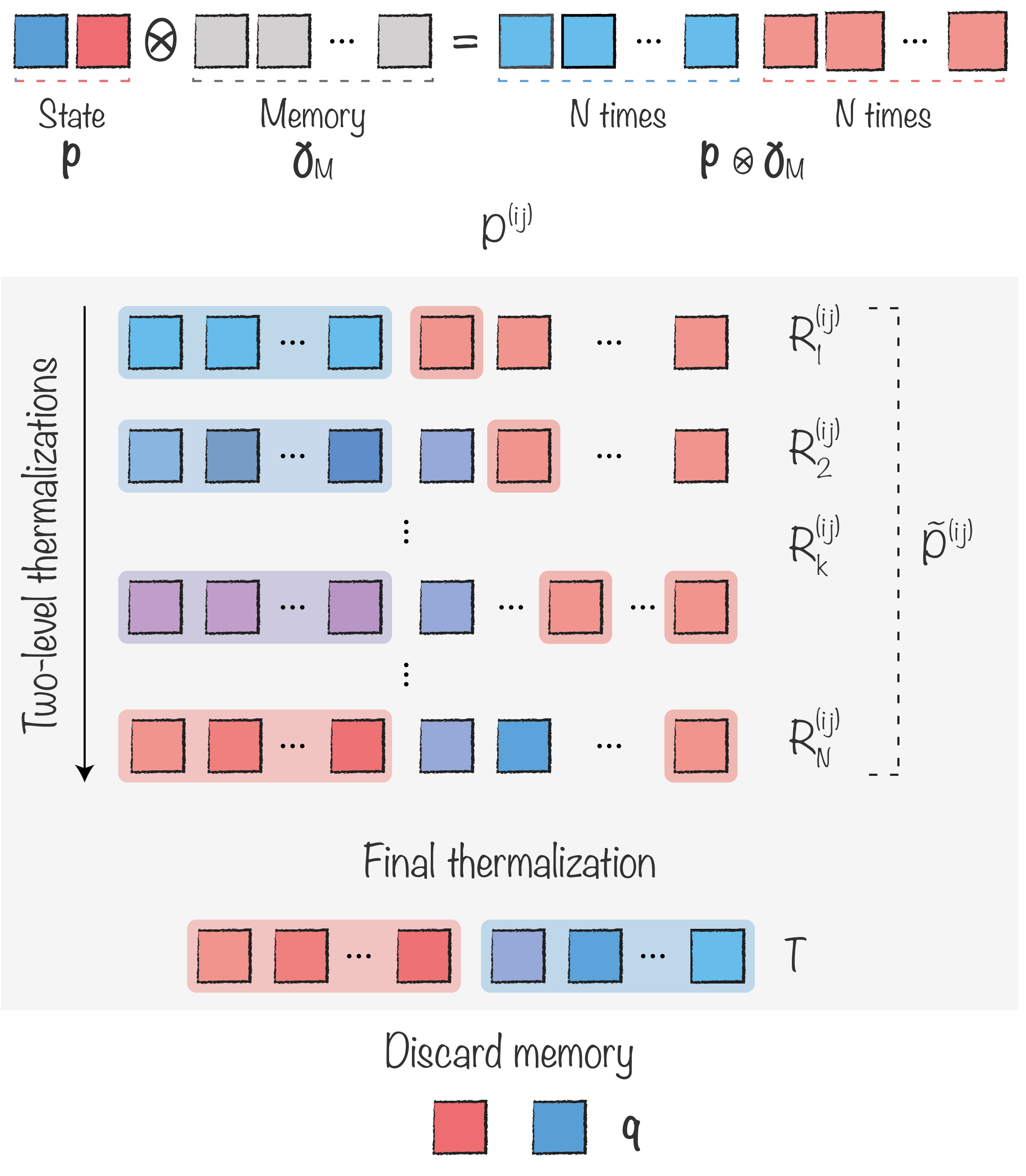
○ Extreme points of $\mathcal{T}_+(\mathbf{p})$: Some are given by non-overlapping β-swaps

Memory-assisted protocols

"We start with 4 glasses of water..."



β - swap protocol



☐ Two-level thermalization: $\{p_i, p_j\} \longrightarrow \frac{p_i + p_j}{\gamma_i + \gamma_j}(\gamma_i, \gamma_j)$

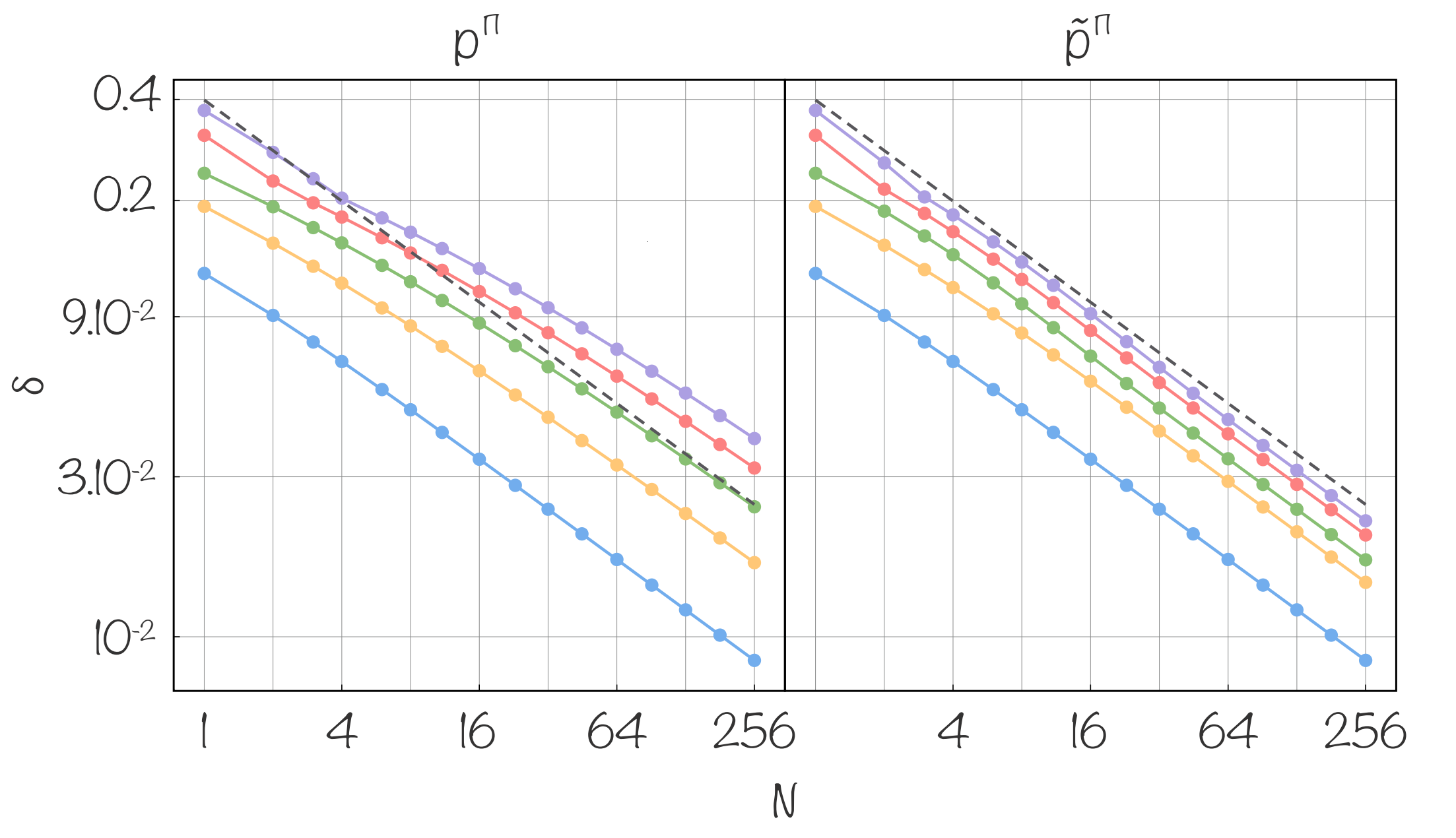
Convergence and conjectures

β = 0

☆ For an N-dimensional memory, the MeMTP protocol act as

$$p^{(ij)}(\mathbf{p}\otimes\boldsymbol{\eta}_M) = \mathbf{q}\otimes\boldsymbol{\eta}_M \text{ , with } \mathbf{q} = [\Pi_{ij} + \varepsilon(\mathbb{1} - \Pi_{ij})]\mathbf{p}$$

and ε given by $\varepsilon = (\pi N)^{-1/2} + o(N^{-1/2})$



? For an N-dimensional memory, $\tilde{p}^\Pi$ gives a better approximation of a permutation Π than p^Π:

$$\delta(\Pi\mathbf{p}, \tilde{\mathbf{q}}) \leqslant \delta(\Pi\mathbf{p}, \mathbf{q})$$

where $\tilde{p}^\Pi(\mathbf{p}\otimes\boldsymbol{\eta}_M) = \tilde{\mathbf{q}}\otimes\boldsymbol{\eta}_M$ and $p^\Pi(\mathbf{p}\otimes\boldsymbol{\eta}_M) = \mathbf{q}\otimes\boldsymbol{\eta}_M$

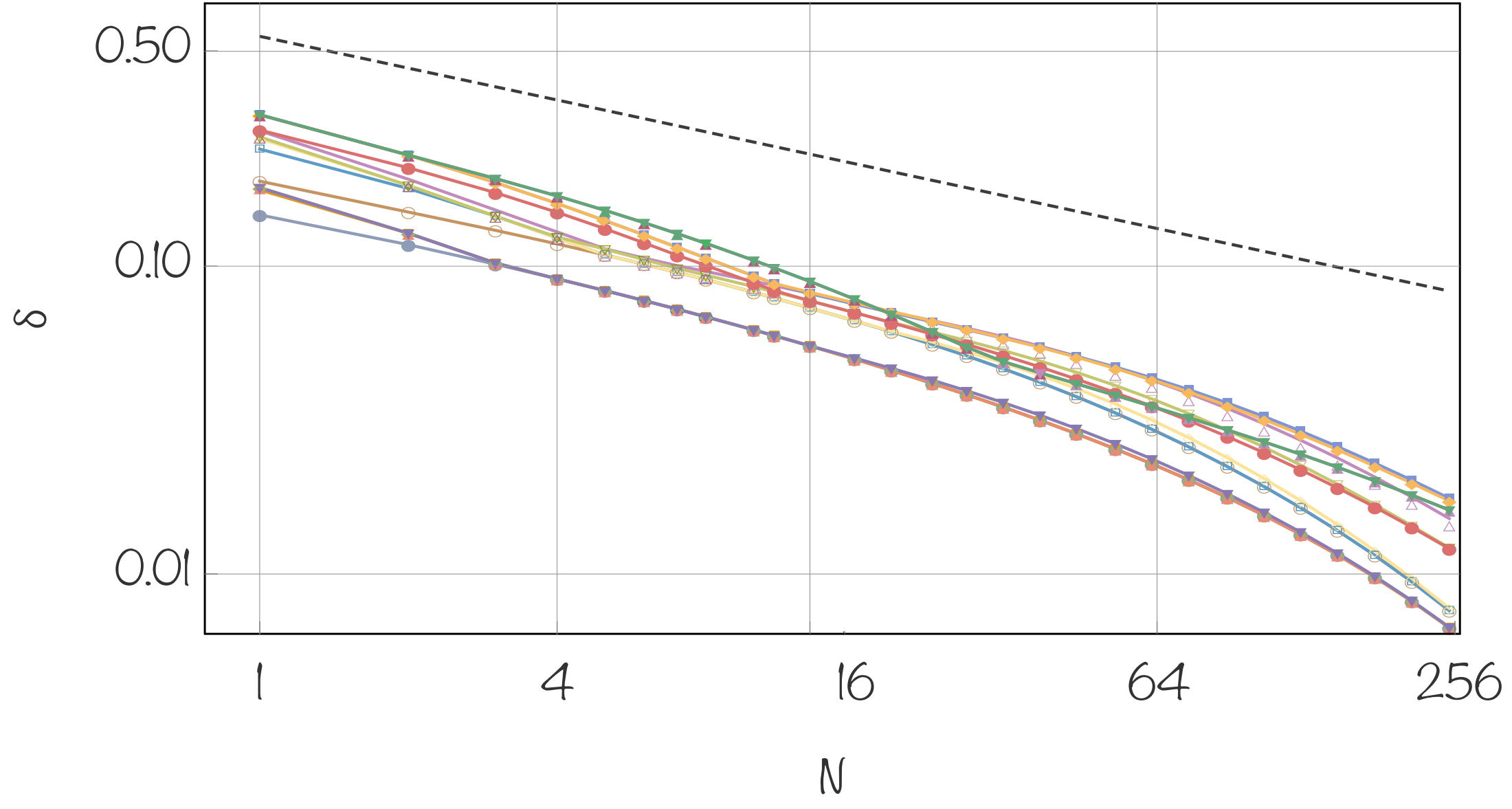
β ≠ 0

☆ For an N-dimensional memory (with trivial Hamiltonian), Π^β_{ij} can be approximated by the MeMTP protocol as

$$p^{(ij)}(\mathbf{p}\otimes\boldsymbol{\eta}_M) = \mathbf{q}\otimes\boldsymbol{\eta}_M$$

with

$$\delta(\mathbf{q},\Pi^\beta_{ij}\mathbf{p}) = \frac{(4\Gamma_i\Gamma_j)^N}{(\Gamma_i - \Gamma_j)^2}\left[\frac{|p_i\Gamma_j - p_j\Gamma_i|}{(N+1)\sqrt{\pi N}} + o(N^{-3/2})\right]$$



? For extreme points p^π with any β-order π:

$$\delta(\mathbf{q}, \mathbf{p}^\pi) = O\left(\frac{e^{-A(\Pi)N}}{N^{-3/2}}\right)$$

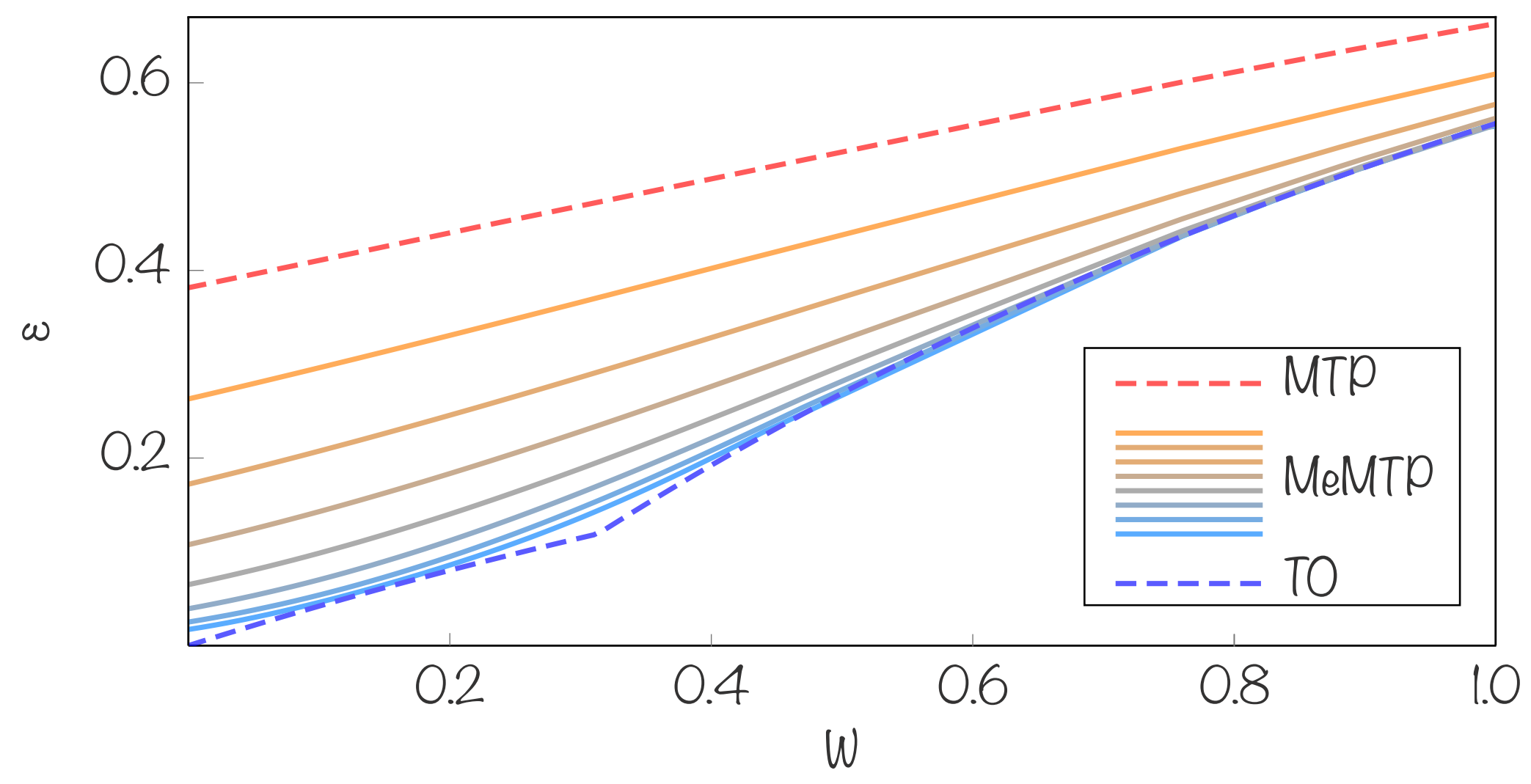
A(Π) = O(1) is a permutation dependent exponent

Applications

Work extraction

☐ Setting: ● ⊗ ▯ $\xrightarrow[\varepsilon]{}$ ●●● ⊗ ▯ (ε-deterministic)

M. Horodecki & Jonathan Oppenheim, Nat.Commun (2013)



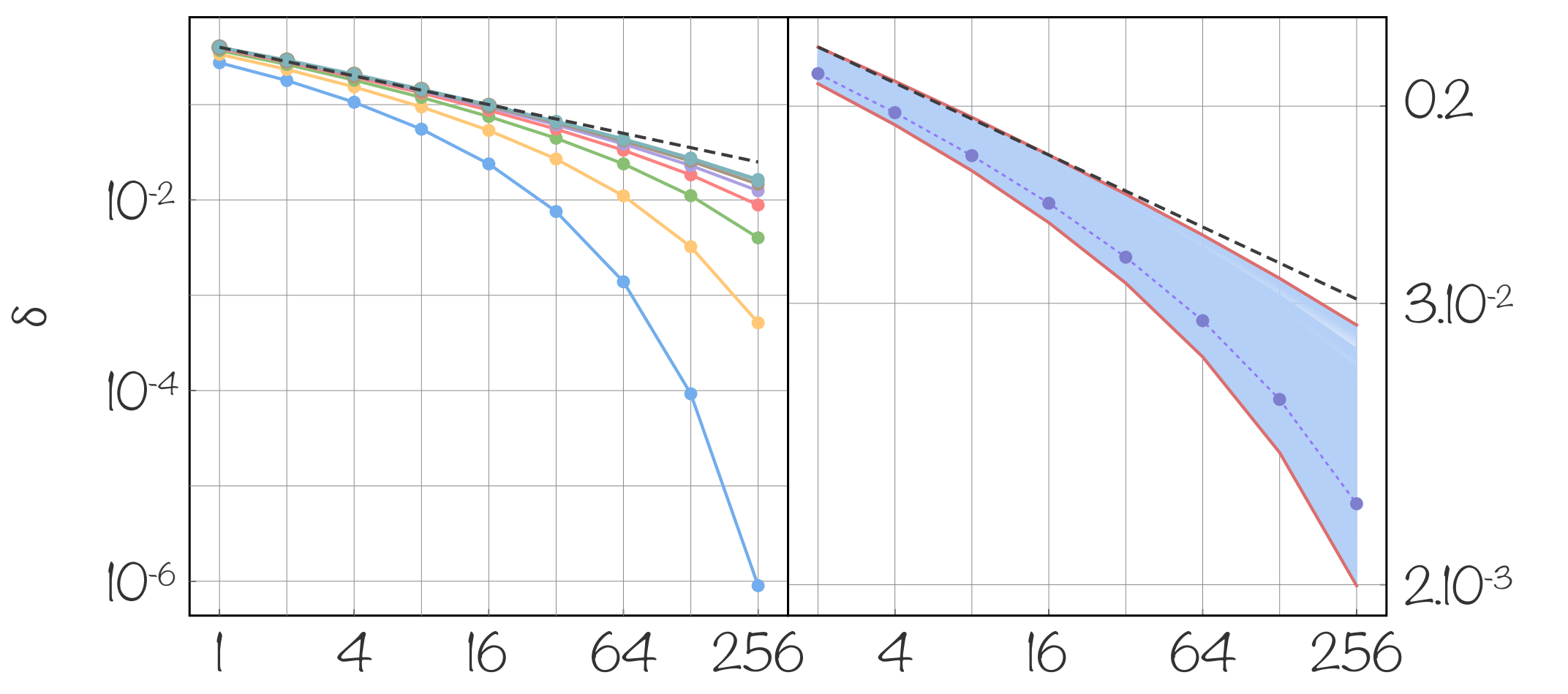
Two-level system with energy splitting Δ prepared in a thermal state at temperature $1/\beta_S$ smaller than the environmental temperature 1/β with parameters $\beta_S\Delta = 2$ and $\beta\Delta = 1$

Two-level control is sufficient for TO

☐ Problem: ● + ●●● $\xrightarrow[\text{thermal operations}]{\text{Elementary}}$ ●

M. Lostaglio, A.M. Alhambra & C. Perry, Quantum (2018)

P. Mazurek & M. Horodecki New. J, Phys (2018)



Our MeMTP is able to transform the initial state into a final state that approximates it arbitrary well!

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P. K. Dick, Total Recall (Vintage Books, 1990)

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