Quantum catalysis in cavity QED

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Outline

1. Introduction
2. Setting the scene
3. Results

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Nicolas Brunner


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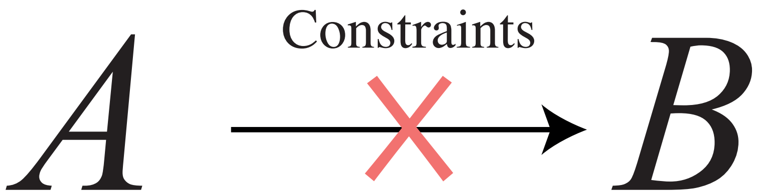
University of Geneva

Based on:
arXiv:2305.19324 - Framework & Applications

Introduction

- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

  $A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.



- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

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correlated

$$A + c \xrightarrow{\text{Constraints}} B + c$$

★ Use **auxiliary degrees of freedom** to **lift** constraints!

- *Resource theories* have uncovered **fundamental** limits and revealed **properties** of *c!*

  highly abstract + limited to special cases.

D. Jonathan and M. Plenio, Phys.Rev.Lett. 83,3566, (1999)

P. Lipka-Bartosik, H. Wilming, and N. H. Y. Ng, arXiv. 2306.00798

Q. Can we go beyond theory and step into **practical** contexts?

Setting the scene

Catalytic picture

Composite system: $\rho_S \otimes \chi_C$

$$\rho_S \rightarrow \sigma_S := \mathrm{Tr}_C[U(\rho_S \otimes \chi_C)U^\dagger]$$

while the state of the catalyst **returns** to its initial state at time τ:

can always be satisfied!

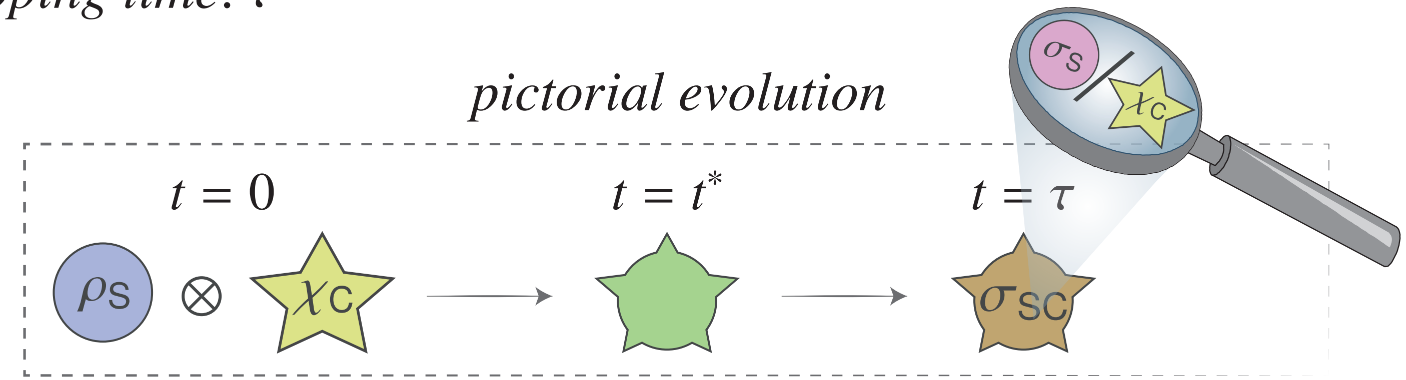
$$\sigma_C := \mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C$$

catalytic constrain

$U(\tau)$

stopping time: τ

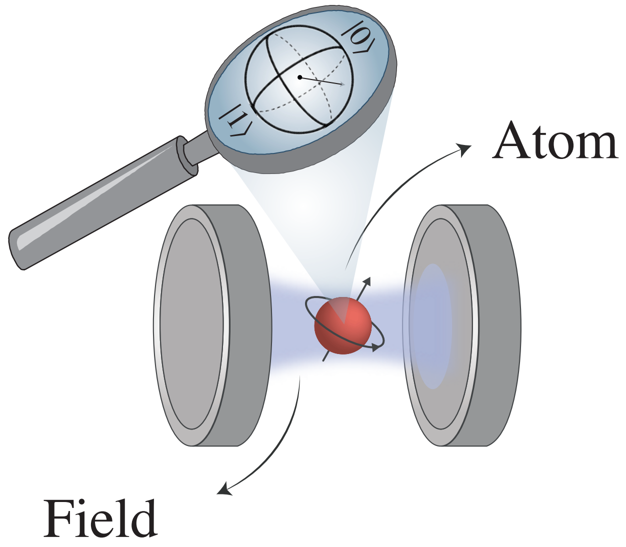
pictorial evolution



Model

Jaynes-Cummings model $(\hbar = 1)$: $H_{\mathsf{SC}} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^\dagger\right)$

only couple pairs of atom-field states: $\{|n+1, g\rangle, |n, e\rangle\}$



So, the eigenproblem is completely determined by $H_{\mathsf{SC}} = \bigoplus_{n=0}^{\infty} H_{\mathsf{SC}}^{(n)}$:

$$H_{\mathsf{SC}}^{(n)} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix} = \begin{bmatrix} (n+1/2)\omega & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1/2)\omega \end{bmatrix} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix}$$

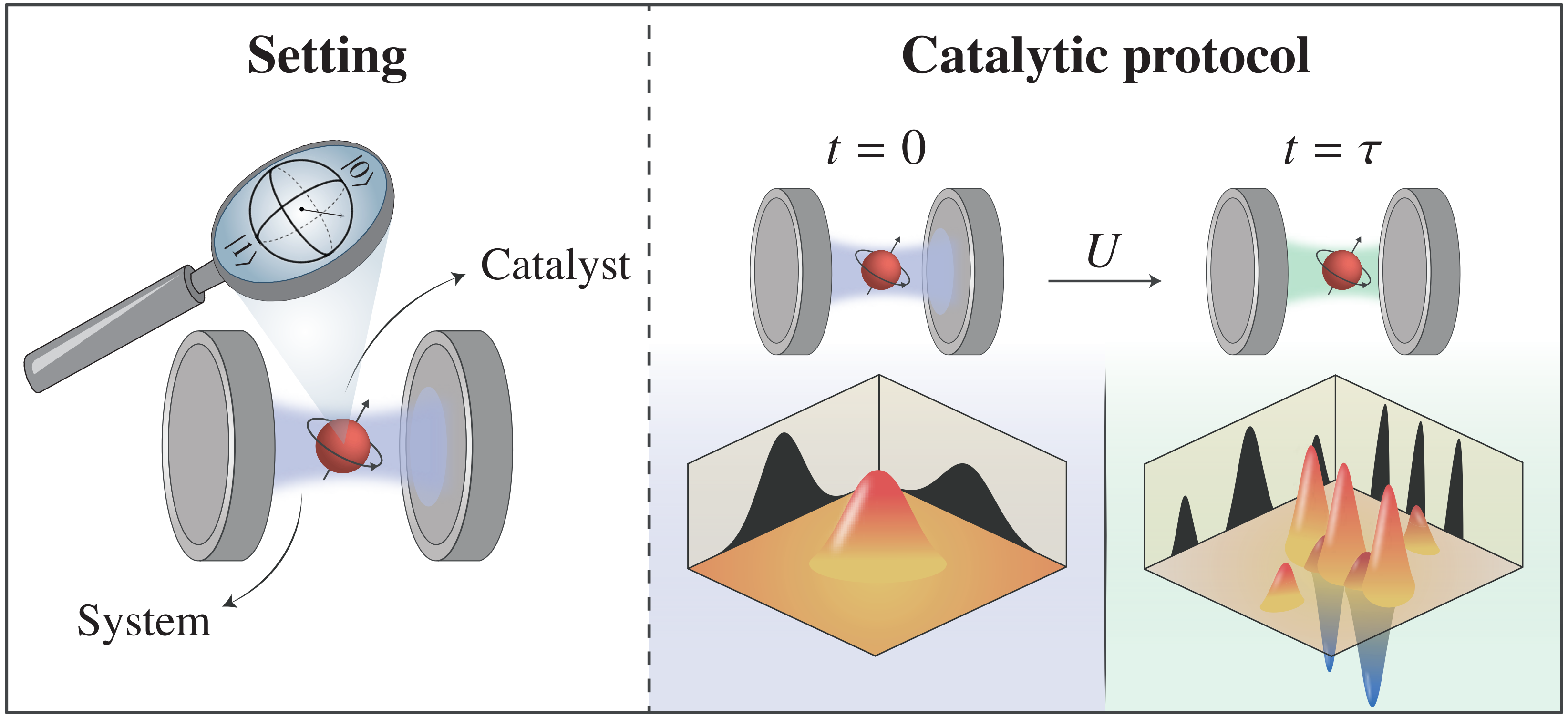
easily diagonalisable

The eigenvalue problem yields the eigenfrequencies: $\omega_\pm^{(n)} = \left(n + \frac{1}{2}\right)\omega \pm 2g\sqrt{n+1}$ → U

$$\sigma_{\mathsf{S/C}} = \mathrm{Tr}_{\mathsf{C/S}}[U(\rho_{\mathsf{S}} \otimes \omega_{\mathsf{C}})U^\dagger]$$

E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).

Spoiler



Q. Which notion of non-classicality?

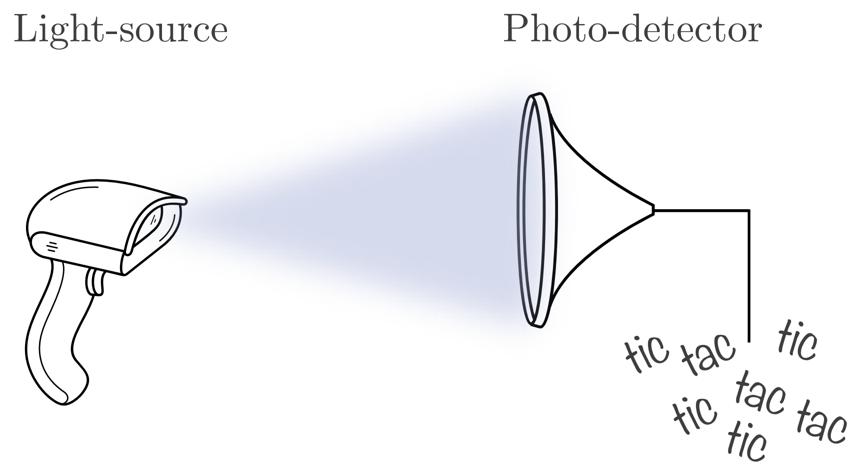
Figures of merit

i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2 \rangle_\sigma - \langle n_{\mathsf{S}} \rangle_\sigma}{\langle n_{\mathsf{S}} \rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

- **Example:**



R. J. Glauber, Phys. Rev. 130, 2529 (1963).

R. Loudon, Phys. Bull. 2721 (1976)

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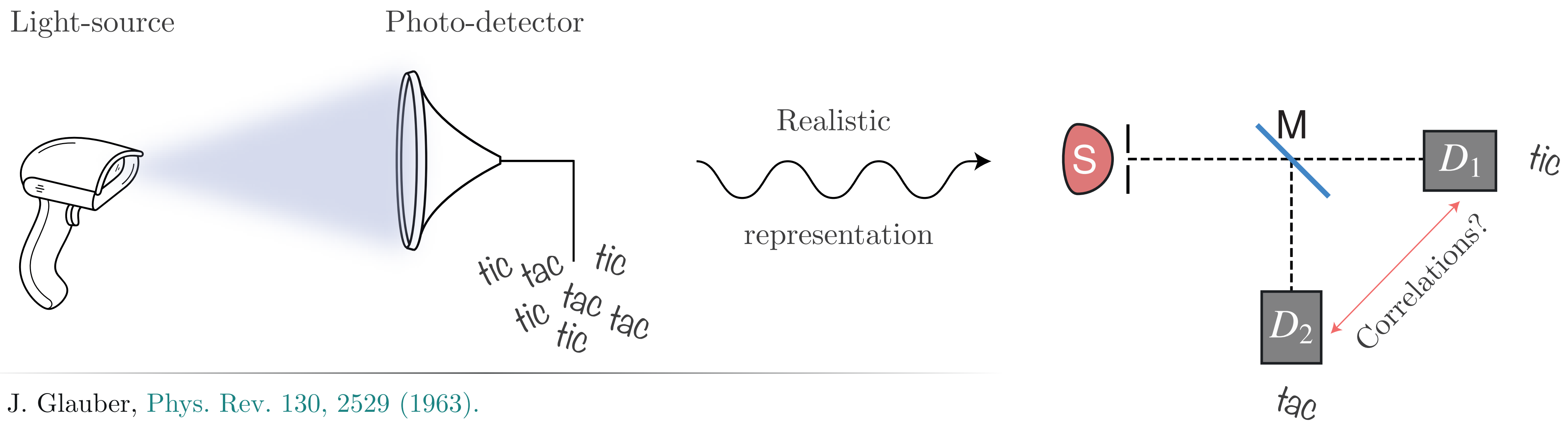

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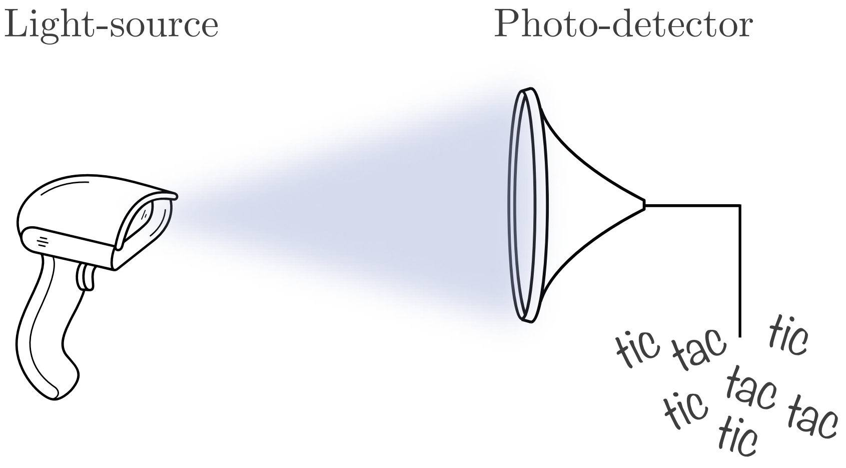
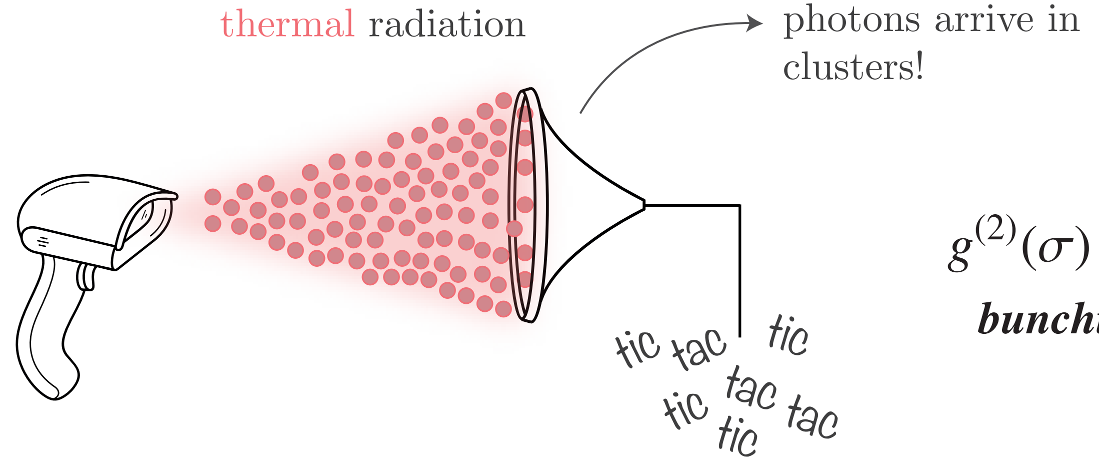


first case →



$g^{(2)}(\sigma) = 2$

bunching

R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

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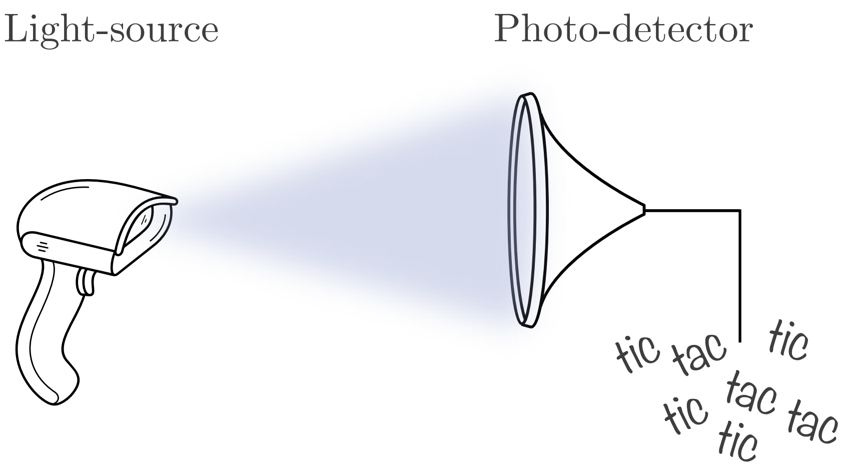
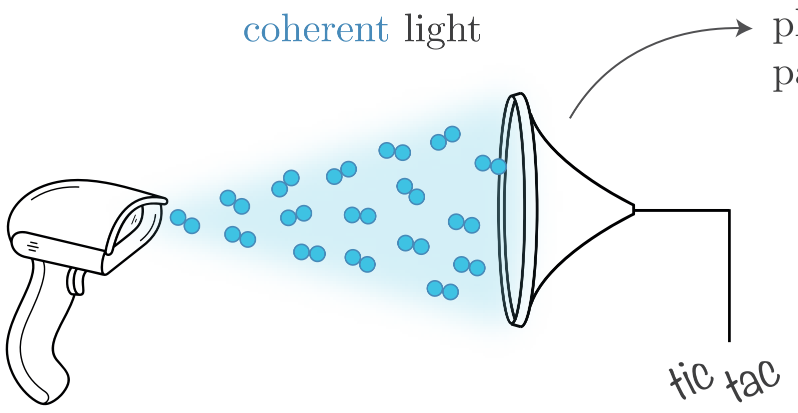


second case →



photons arrive in pairs!

$$g^{(2)}(\sigma) = 1$$

R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

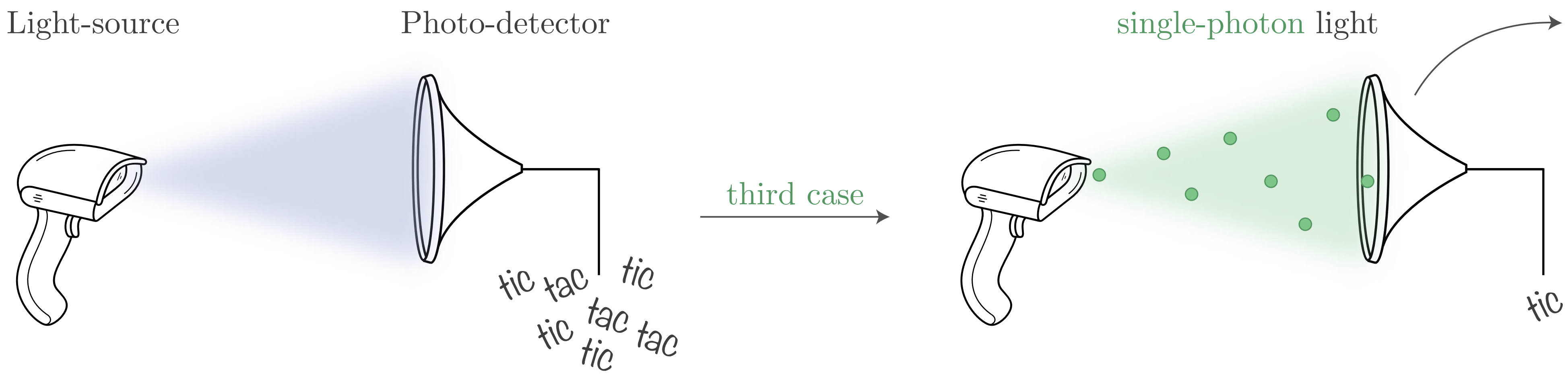
Figures of merit

i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_S^2 \rangle_\sigma - \langle n_S \rangle_\sigma}{\langle n_S \rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

○ **Example:**



photons are never detected simultaneously!

$g^{(2)}(\sigma) < 1$

anti-bunching

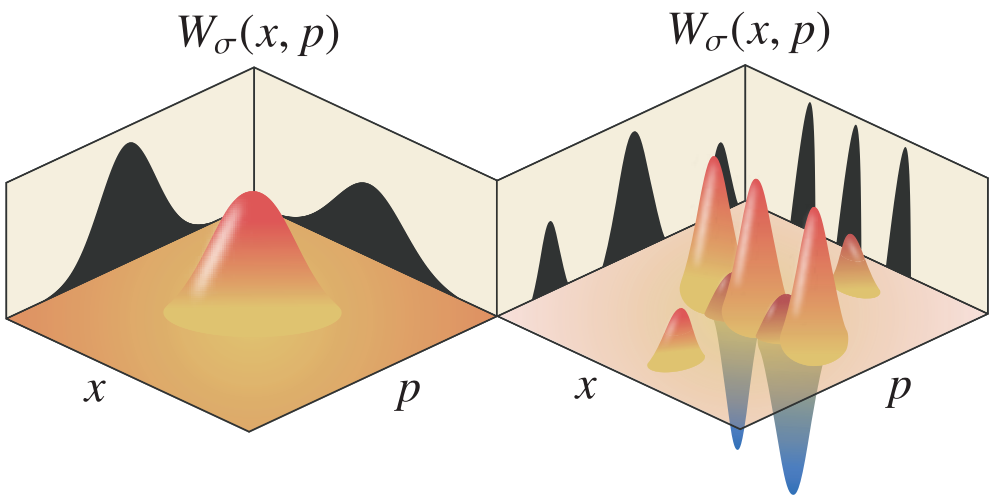
R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

ii. Wigner function:

$$W_\sigma(x,p) = \frac{1}{\pi}\int e^{2ipx'}\langle x - x'|\sigma|x + x'\rangle\,\mathrm{d}x'$$

- **Example:**



Q. How to quantify the degree of non-classicality?

$$\mathsf{W}(\sigma) := \log\left(\int \mathrm{d}x\,\mathrm{d}p\;|W_\sigma(x,p)|\right)$$

E. Wigner, Phys. Rev. 40, 749 (1932)

A. Kenfack and K. Życzkowski, J. Opt., B Quantum Semiclass. Opt. 6, 396 (2004)

F. Albarelli, M G. Genoni, M G. A. Paris, A. Ferraro, Phys. Rev. A 98, 052350 (2018)

Results

Statement of the problem

Task: generation of non-classical light in a catalytic way.

Consideration: $\rho_{\mathsf{S}} = |\alpha\rangle\langle\alpha|$ where

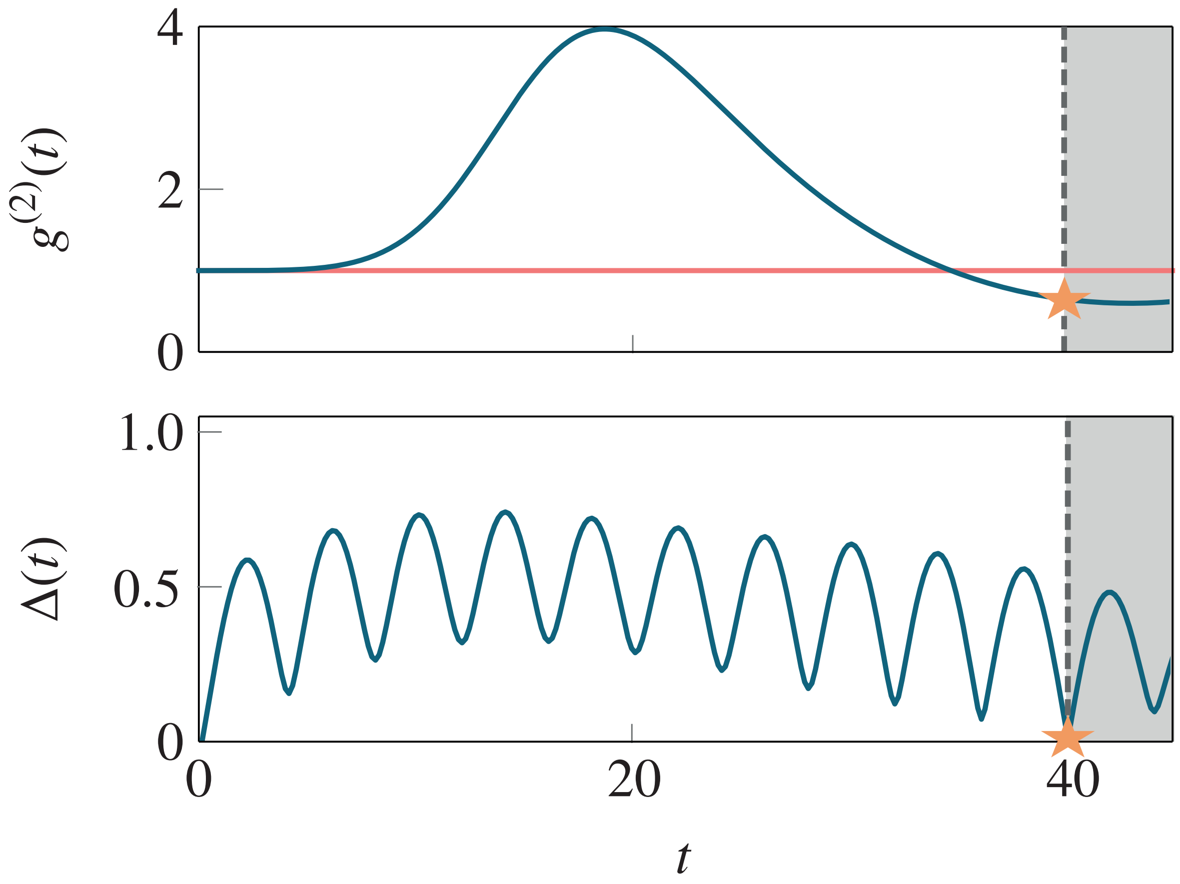
$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} |n\rangle \,.$$

→ $g^{(2)}(\rho_{\mathsf{S}}) = 1$ and $\mathsf{W} = 0$

Goal: find χ_{C} and τ, such that $\mathrm{Tr}\Big[U(\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}})U^\dagger\Big] = \chi_{\mathsf{C}}$.

First example

Generating light with **sub-Poissonian** photon statistics:



Definitions

$$g^{(2)}(t) := g^{(2)}[\sigma_S(t)]$$

$$\Delta(t) := \|\chi_C - \sigma_C\|_1$$

Parameters

$$\alpha = 1/\sqrt{2}$$

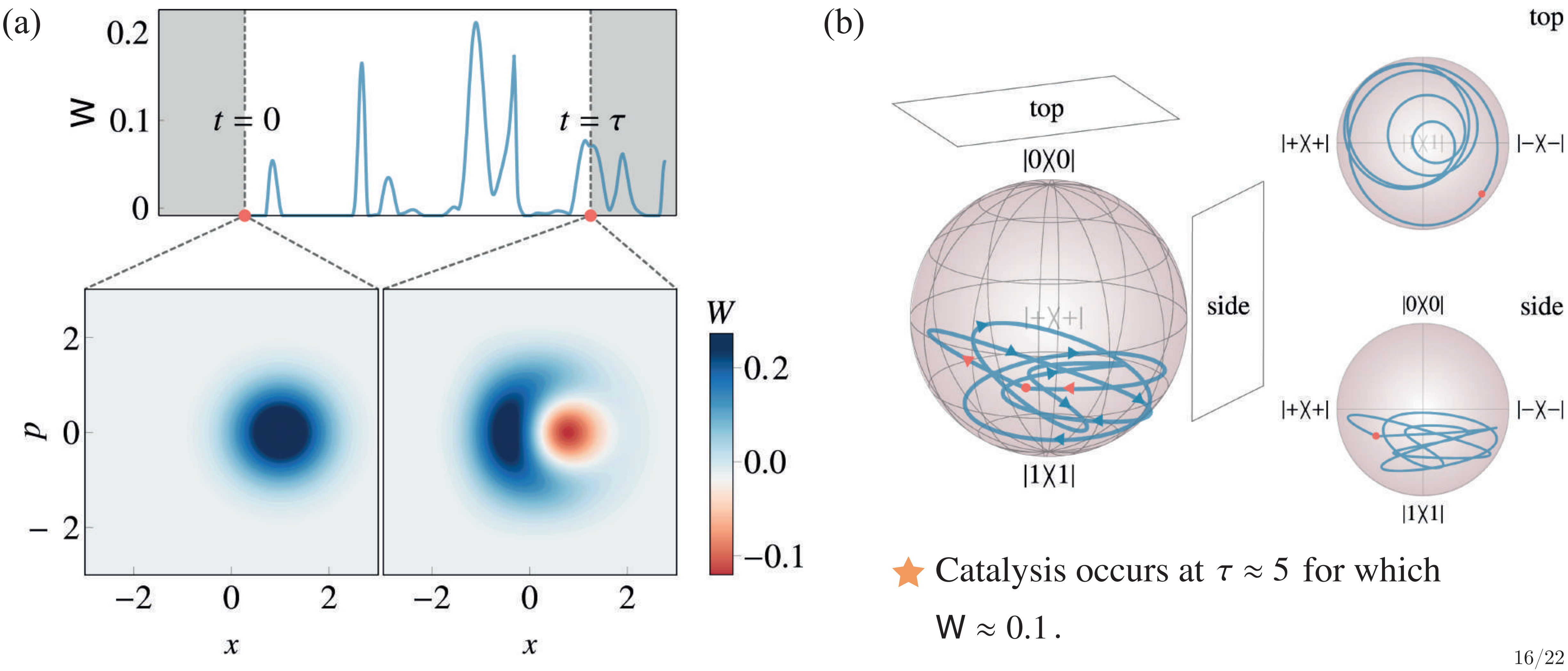
$$\omega = 2\pi$$

$$g = \pi$$

★ Catalysis occurs at $\tau \approx 40$ for which $g^{(2)}(\tau) \approx 0.5$.

Second example

Generating light with **negative** Wigner function:



★ Catalysis occurs at $\tau \approx 5$ for which $\mathsf{W} \approx 0.1$.

Mechanism of catalysis

- **Observation 1:** Energy-preserving $[U(t), n_{\mathsf{S}} + n_{\mathsf{C}}] = 0$.

- **Observation 2:** Catalytic constrain $\longrightarrow$ all moments of n_{C} are preserved and $\langle n_{\mathsf{S}}\rangle_\rho = \langle n_{\mathsf{C}}\rangle_\sigma$.

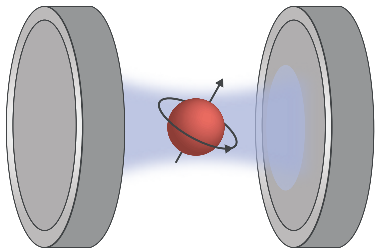
$$\langle n_{\mathsf{S}}^2\rangle_\sigma = \langle n_{\mathsf{S}}^2\rangle_\rho + 2\Big(\langle n_{\mathsf{S}}\rangle_\sigma\langle n_{\mathsf{C}}\rangle_\sigma - \langle n_{\mathsf{S}} \otimes n_{\mathsf{C}}\rangle_\sigma\Big)$$

measure of correlations

- **Observation 3:** The second moment changes only when the **system** and **catalyst** become **correlated**.

Q. Which **atomic states** lead to catalyst?

Which states lead to catalyst?



General case →

$$\rho_S = \sum_{n,m}^{\infty} p_{n,m} |n\rangle\langle m| \quad , \quad \chi_C = q\,|g\rangle\langle g| + r\,|g\rangle\langle e| + r^*\,|e\rangle\langle g| + [1-q]\,|e\rangle\langle e|$$

Recall:

$$\mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C$$

Follows that the ground-state occupation can be decomposed as $q = q_{\rm inc} + q_{\rm coh}$:

$$q_{\rm inc} = \frac{1}{Q}\sum_{n=0}^{\infty} p_n s_n^2 \quad \text{and} \quad q_{\rm coh} = \frac{1}{Q}\sum_{n=0}^{\infty} y_n$$

$p_n \to p_{n,n}$

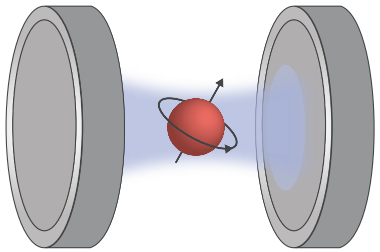
$$Q := \sum_{n=0}^{\infty} (p_n + p_{n+1}) s_n^2$$

$$s_n := \sin\left(gt\sqrt{n+1}\right)$$

$$y_n := 2\,\mathrm{Im}\,[r p_{n+1,n}] s_n c_n$$

$$c_n := \cos\left(gt\sqrt{n+1}\right)$$

Which states lead to catalyst?



General case

$$\rho_S = \sum_{n,m}^{\infty} p_{n,m} |n\rangle\langle m| \quad , \quad \chi_C = q\,|g\rangle\langle g| + r\,|g\rangle\langle e| + r^*\,|e\rangle\langle g| + [1-q]\,|e\rangle\langle e|$$

Recall:

$$\mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C$$

where the off-diagonal term r satisfies:

$$r = \frac{i(a_3 a_4^* + a_1^* a_4)}{|a_1|^2 - |a_3|^2} - \frac{i(a_3 a_2^* + a_1^* a_2)}{|a_1|^2 - |a_3|^2} q$$

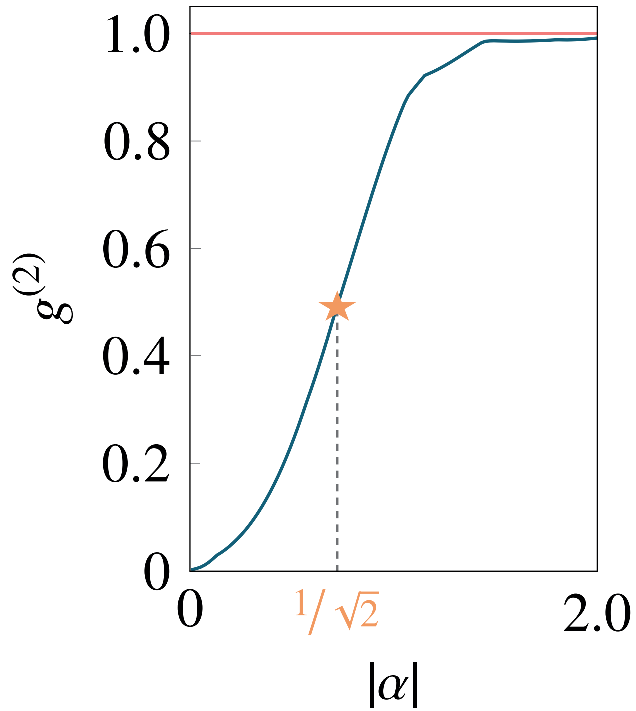
with

$$a_1 = \sum_{n=0}^{\infty} p_{n,n} c_{n-1} c_n - e^{-i\omega\tau}, \qquad a_3 = \sum_{n=0}^{\infty} p_{n,n+2} s_n s_{n+1},$$

$$a_2 = \sum_{n=0}^{\infty} p_{n,n+1} s_n [c_{n-1} + c_{n+1}], \qquad a_4 = \sum_{n=0}^{\infty} p_{n,n+1} s_n c_{n+1}$$

How general is catalysis?

Q. How often a catalytic evolution leads to a non-classical state?



Parameters

$\omega = 2\pi, g = \pi, g\tau \leq 100$

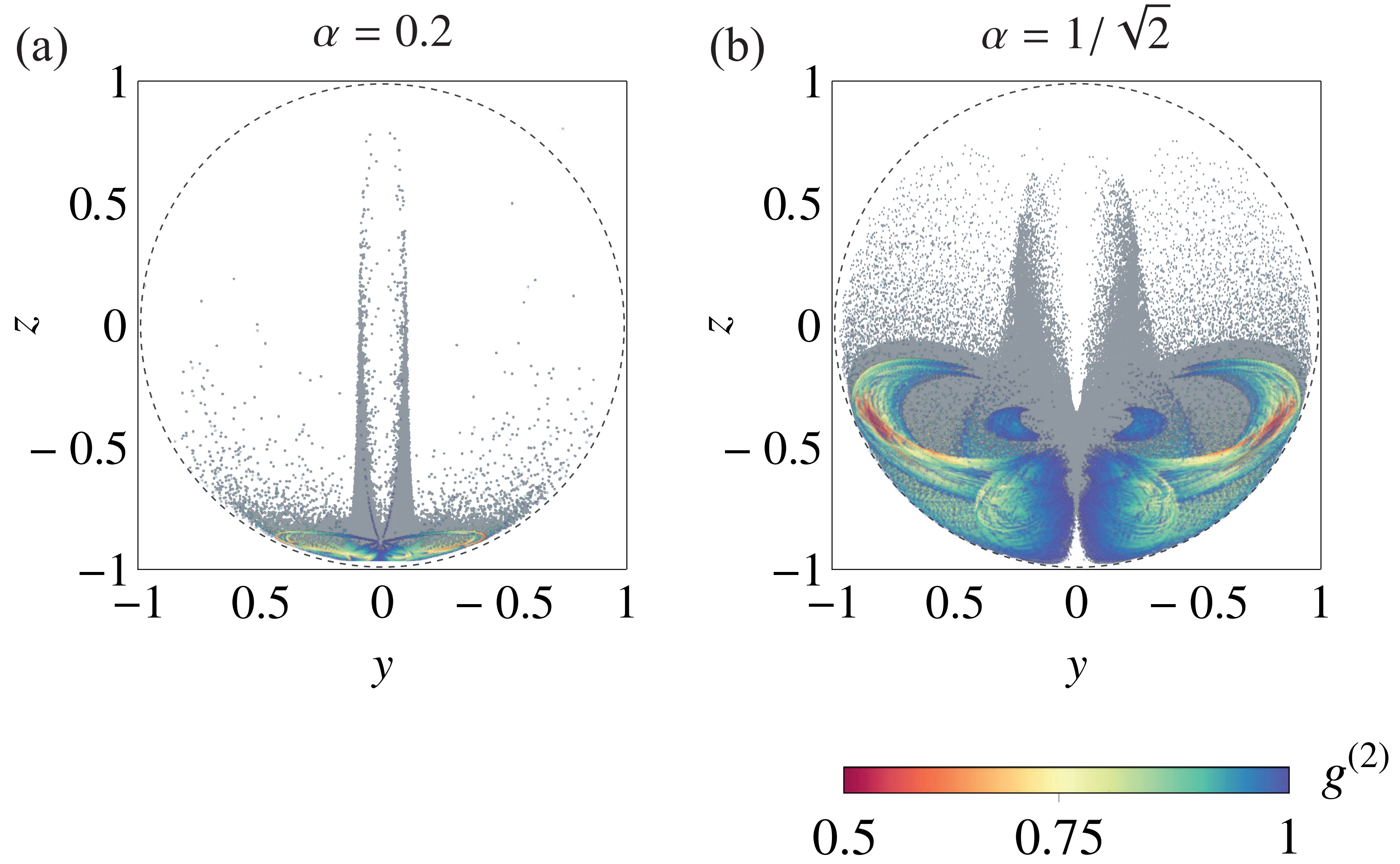
$$g^{(2)}(\sigma_{\mathsf{S}}) = g^{(2)}(\rho_{\mathsf{S}}) - \frac{2}{\langle n_{\mathsf{S}} \rangle_\rho^2} \left[\langle n_{\mathsf{S}} \otimes n_{\mathsf{C}} \rangle_\sigma - (1-q) \langle n_{\mathsf{S}} \rangle_\rho \right]$$

where

$$\langle n_{\mathsf{S}} \otimes n_{\mathsf{C}} \rangle_\sigma = \sum_{n=0}^{\infty} n \left[(1-q) p_n c_n^2 + y_n + q p_{n+1} s_n^2 \right]$$

How general is catalysis?

Q. How does the set of catalytic states look like? Highly non-trivial!



Outlook

Summary

! Catalytic process in a paradigmatic quantum optics setup:

- Generation of non-classical states of light.
- Mechanism behind the catalytic evolution.
- Atomic states.

What's next?

? Change the role between main system and catalyst.

? Different models.

? Go beyond quantum optics.

Thank you!