When to/not to quantumly simulate a classical transition?

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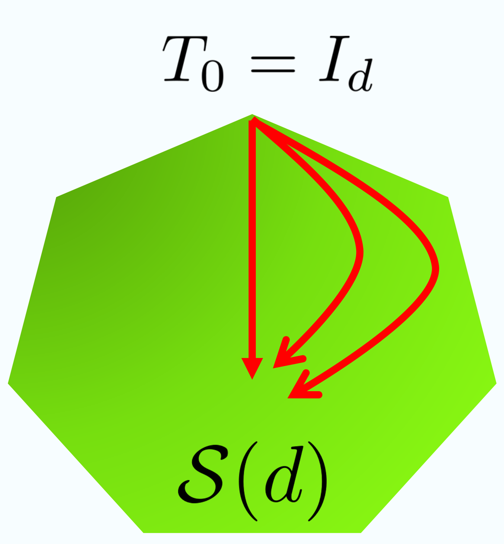
A. Abstract:

The concept of Markovianity, asking whether a given channel can be realised through a memoryless interaction with the environment, has been a long-standing question in both classical and quantum physics. Over the years, various applications of Markovian maps have been introduced separately for classical and quantum systems. Recently, the novel concept of quantum embeddability has been discovered, which constructs a bridge between the quantum Markovian world and the classical realm. A stochastic matrix is said to be quantum embeddable if it is the classical action of a Markovian quantum channel. This allows the benefits of memoryless quantum evolution to be applied to classical systems that typically require memory to be simulated. Here, we elaborate on which classical transition matrices are memory-wise beneficial to be simulated quantumly and when we still need memory anyway.

B. Introduction:

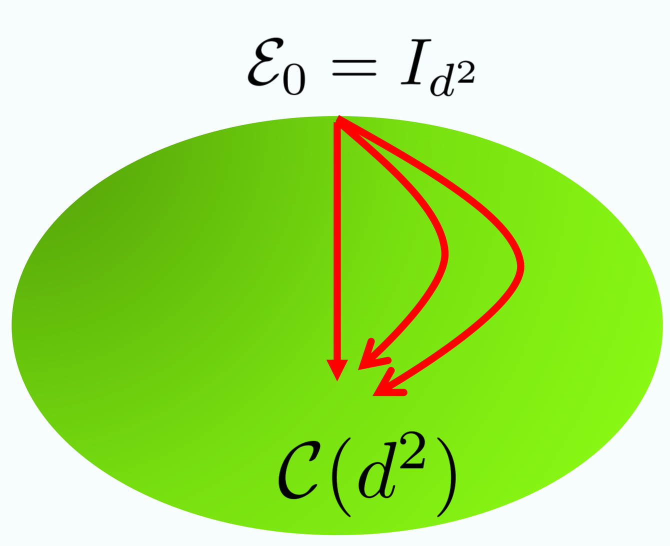
Classical Markovianity

A stochastic matrix $T \in \mathcal{S}(d)$ is called Markovian if $T = \mathrm{e}^{L}$, where L is a Kolmogorov operator. That is, T is Markovian if it lies on a trajectory generated by a memoryless interaction.



Quantum Markovianity

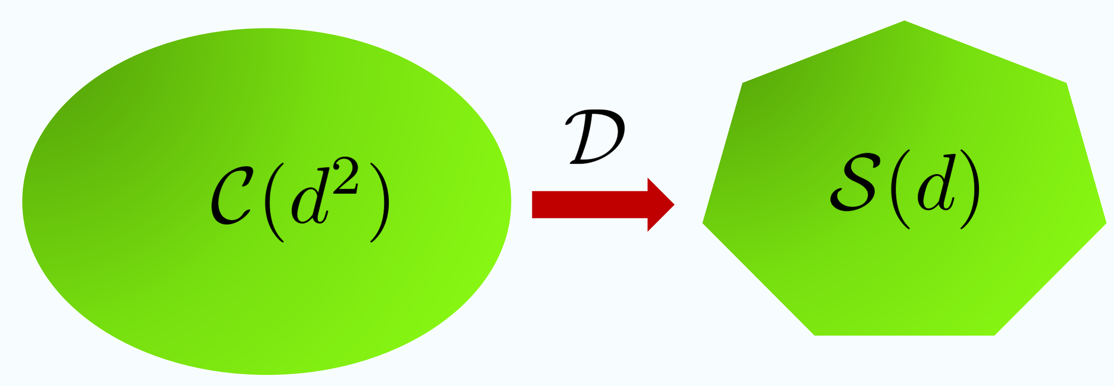
A quantum channel $\mathcal{E} \in \mathcal{C}(d^2)$ is called Markovian if $\mathcal{E} = \mathrm{e}^{\mathcal{L}}$, where $\mathcal{L}$ is a Lindblad generator. That is, $\mathcal{E}$ is Markovian if it lies on a trajectory generated by a memoryless interaction.



Simulation Map

A stochastic matrix $T \in \mathcal{S}(d)$ is the classical action of a quantum channel $\mathcal{E} \in \mathcal{C}(d^2)$ through

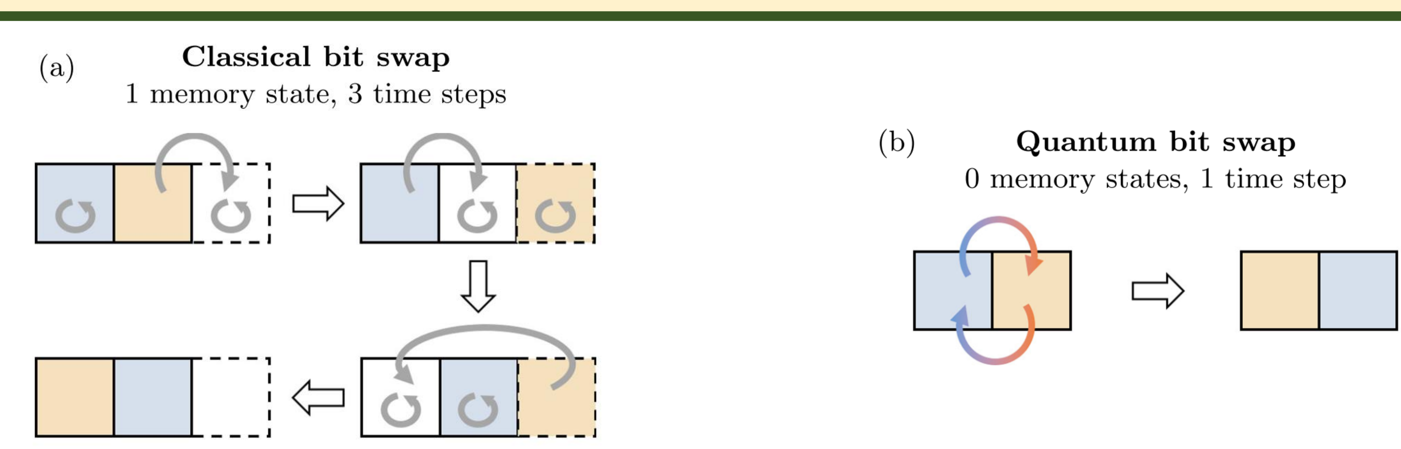
$$T_{ij} = \langle i|\mathcal{E}(|j\rangle\langle j|)|i\rangle \implies T = \mathcal{D}(\mathcal{E})$$



Markovianity is an NP-hard problem due to non-uniqueness of Logarithm, also the simulation map is typically highly non-injective.

C. The Problem and Motivation:

Which stochastic matrix T is the classical action of some Markovian quantum channel? Such a matrix T is called Quantum Embeddable, and can be quantumly simulated with memory advantage.



D. Results:

. Embeddable/non-embeddable maps in dimension d:

1. Every classical Markovain stochastic matrix is quantum embeddable, i.e., if L exists, then $\mathcal{L}$ exists.

2. All unistochastic matrices are quantum embeddable. $T_{ij} = |U_{ij}|^2 \implies \mathcal{E}(\cdot) = U(\cdot)U^\dagger$

3. T is embeddable if it has the block form $R_1 \oplus R_2$ where R_1 is classical Markovian and R_2 is unistochastic.

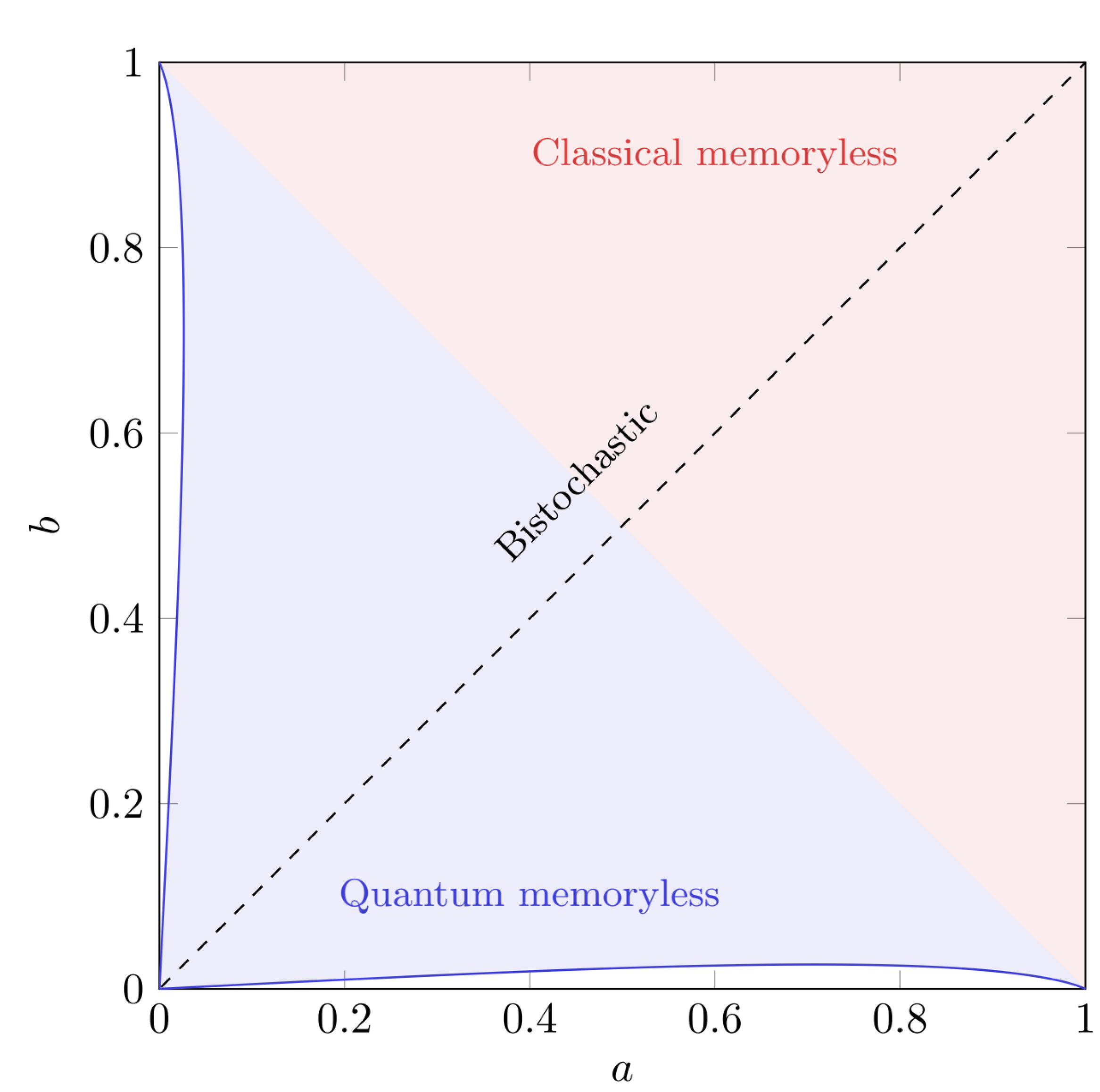
4. T is embeddable if it has the block form $R_1 \oplus R_2$ where R_1 and R_2 are quantum embeddable.

5. T is quantum embeddable if it has the following block form $\left(\begin{array}{c|c} R & S \\ \hline A & B \end{array}\right)$, where R is quantum embeddable and the columns of S are a copy of some columns of R.

6. T is not embeddable if there exists a set of indices $\mathcal{I} \subset \{1, \ldots, d\}$ with $i_0 \in \mathcal{I}$ such that for all $i \in \mathcal{I}$ we have $T_{i_0 i} = 1$ and there exists an index k in the complementary set of $\mathcal{I}$ with $\sum_{i \in \mathcal{I} \setminus \{i_0\}} T_{ik} = 1$.

. Maps of dimension 2

$$T = \begin{pmatrix} a & 1-b \\ 1-a & b \end{pmatrix},$$



E. References:

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