Some valleys in a landscape of (quantum) computation

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Overview

The space of possible quantum computations forms a complicated high dimensional "landscape", we think of the height of a point in this landscape as representing how difficult a particular computation is to simulate classically.

Fig. 1: Landscape from the Tatra Mountains (circa 1903) Jan Stanisawski

We have explored some "valleys" in this landscape - classes of computation which are relatively easy to simulate. Here are some examples

1. Clifford + T [1, 3]
2. Bounded qubit count computations
3. Bounded entangling gate count
4. Matchgate+Magic*
5. Fermionic linear optic+Magic*

∗ These two classes are identical, but their classical simulation is relatively unexplored.
Almost nothing is known about "mountains" in this landscape - places where classical simulation is superpolynomially expensive. It is widely believed that there are mountains, but proving it would prove $\mathrm{P} \neq \mathrm{NP}$ (and resolve a millennium problem).

Probability estimation overview

This is joint work with Hakop Pashayan, Kamil Korzekwa and Stephen Bartlett. We came up with a classical algorithm to solve problem (1).

Problem (1)
Given a quantum circuit U composed of CX, Hadamard and arbitrary single qubit phase gates, an input computational basis state $|x\rangle$, an output computational basis state $|y\rangle$ define the Born-rule probability

$$P = ||\langle y| U |x\rangle||^2. \tag{1}$$

Given the inputs above and two real numbers ε and δ return a number P drawn from a probability distribution such that

$$\Pr(|P - p| > \varepsilon) < \delta. \tag{2}$$

Our algorithm consists of two main sub-algorithms, Compute and Estimate, as well as a polynomial-time algorithm called Compress which decides which of Compute or Estimate will be more efficient.

Compress
Compress outputs a smaller quantum circuit as well as an integer r. One of the most interesting feature of our work is the discovery of a new class of efficiently simulable quantum computations - namely those for which r is close to the number of non-Clifford phase-gates t.

Probability estimation performance

Compute
Compute exactly computes a Born rule probability with runtime

$$\tau_{\text{compute}} = \tilde{O}(2^{t-r}t), \tag{3}$$

where the notation $\tilde{O}$ hides additative polynomial terms in t and n (the number of qubits) and r is the integer output by the Compress algorithm.

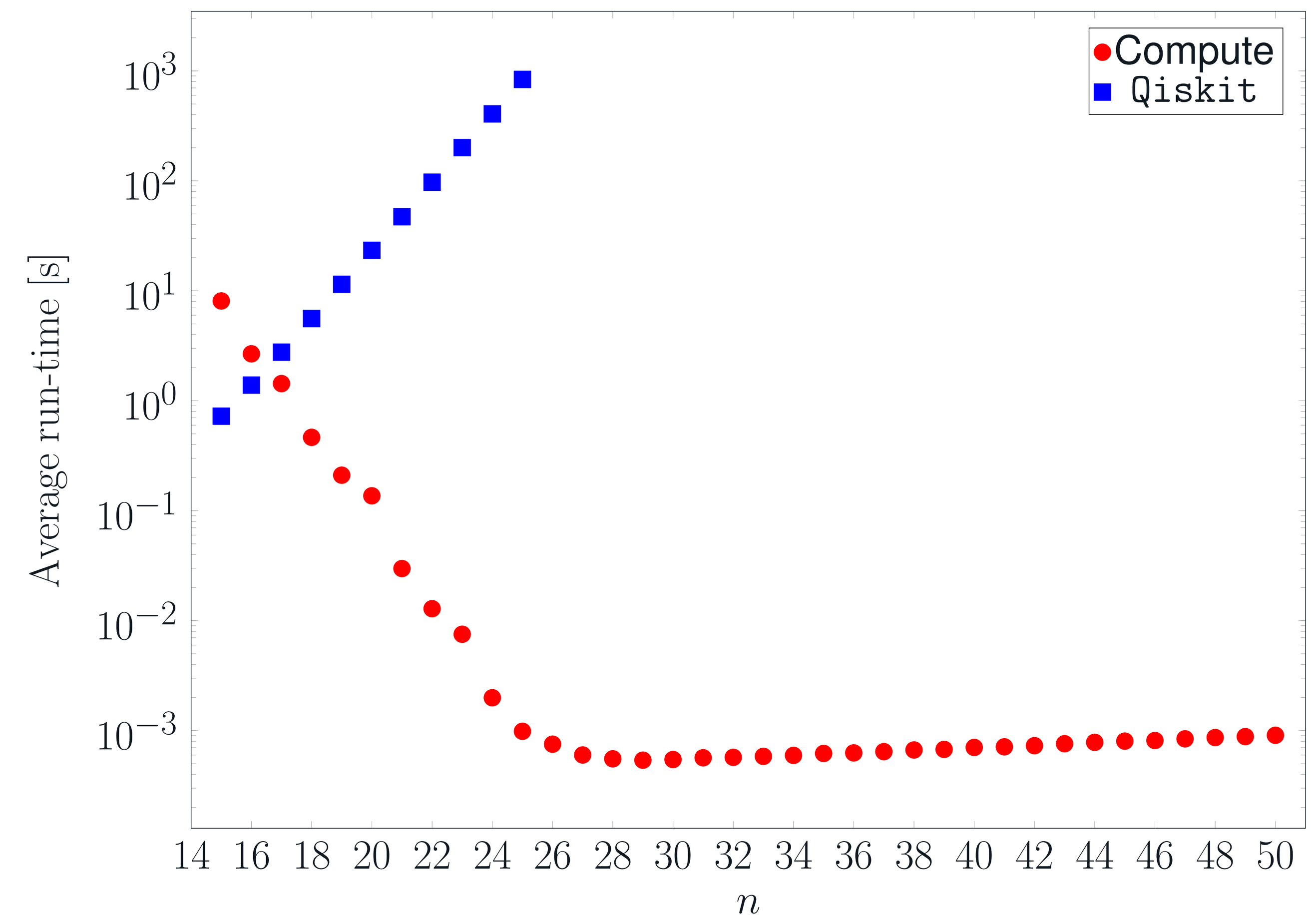

Fig. 2: Performance of the Compute algorithm for random circuits. These circuits were generated with 10^5 basic Clifford gates, 80 $T_{\frac{1}{4}}$ gates and 20 measured qubits. The averages are over 100 sampled circuit with the same circuits given to Qiskit's statevector simulator[2] and the Compute algorithm. The runtime of Compute is exponential in $t - r$ where $r \approx n - w$ while the runtime of the statevector simulator is exponential in n.

Estimate
Estimate works best for circuits where r is very small (exactly the opposite of Compute). It is the most complicated of our algorithm and is state of the art for problem (1). Its performance is exponential in the number of non-Clifford gates t, logarithmic in δ and polynomial in all other parameters.

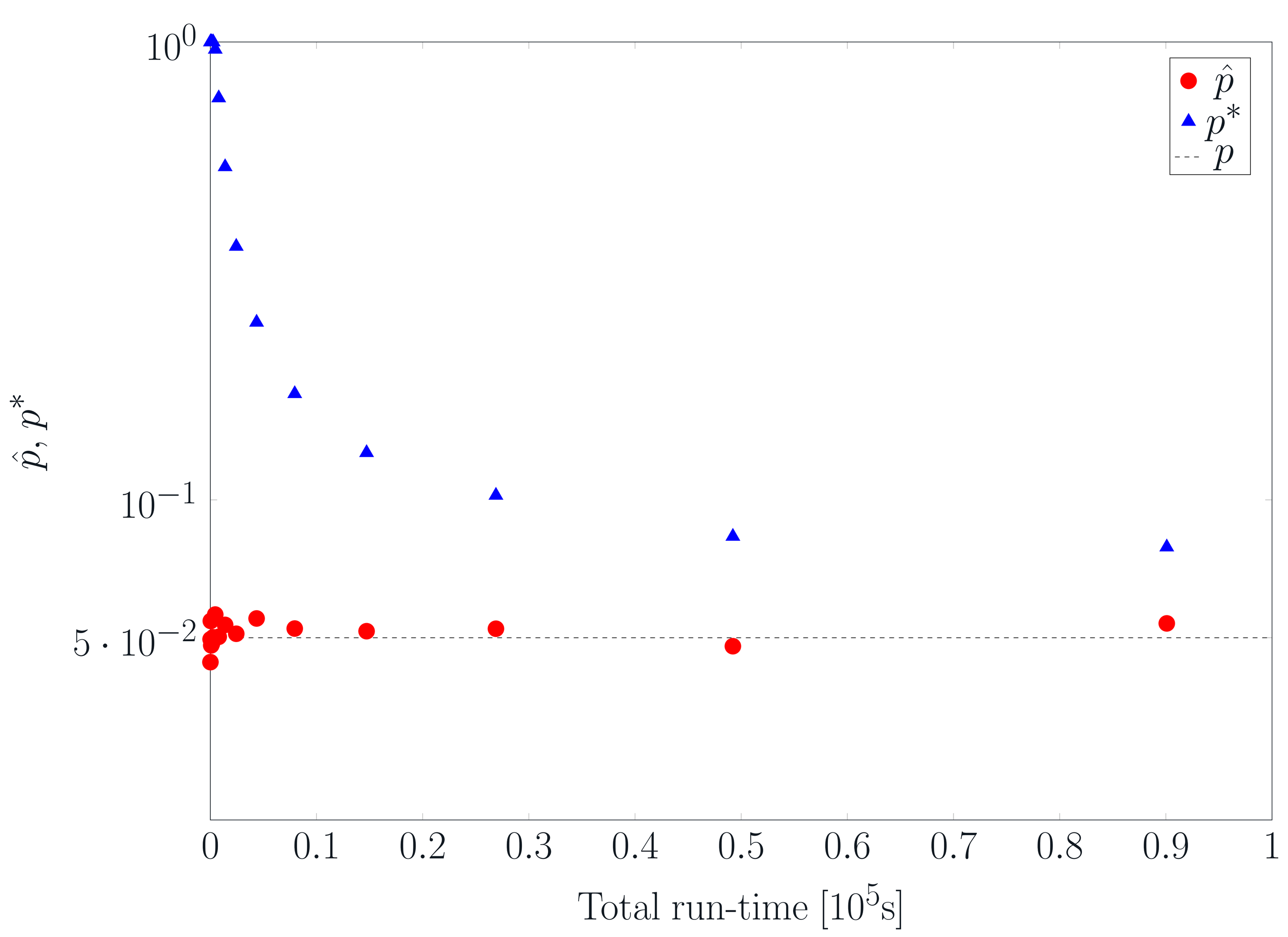

Fig. 3: Performance of the Estimate algorithm for an $n = 50$ qubit, $t = 60$ non-Clifford gate circuit constructed with a stabilizer extent $\xi^* \approx 3767$ equal to that of 52 $T_{\frac{1}{4}}$ gates. The Circuit consisted of 60 single qubit diagonal non-Clifford gates and approximately 2000 basic Clifford gates. The first $w = 8$ qubits were measured and the projector rank was $r = 10$. Each call to the RawEstim subprocedure provides an estimate $\hat{p}_k$ (red circles) and an upper bound $p_k^* = \hat{p}_k + \varepsilon_k$ (blue triangles) of the target Born rule probability p and incurs a time cost of $\tau_k = 2^k\tau_0$.

Fermionic linear optics+Magic

We begin with two facts

1. Fermionic linear optics circuits acting on Gaussian states may be simulated in polynomial time [5]
2. Fermionic linear optics circuits acting on certain "magic" non-Gaussian states are universal for quantum computation[4].

This situation is exactly analagous to the case of the Clifford/stabilizer subtheory. It turns out that it is possible to write the "magic" non-Gaussian states as sums of Gaussian states in two different ways.

1. As a statevector the magic states are each a superposition of *two* Gaussian states
2. As a density matrix the magic states are each a sum of *sixteen* Majorana Fermion operators

This suggests two simulation algorithms for universal quantum circuits which are highly efficient for circuits which are close to Gaussian ones, essentially mirroring the statevector vs density matrix divide for simulation of Clifford+T circuits.
In each case one "gadgetizes" the non-Fermionic circuit to obtain a Fermionic circuit acting on non-Fermionic inputs, then expresses the non-Fermionic inputs as an (exponentially large) sum over Fermionic inputs, before applying a sampling algorithm in a simular way to those already known from Clifford-based simulators.
I am working on several open problems must be solved in order to complete this program.

Links

- Paper on arXiv - https://arxiv.org/abs/2101.12223
- Code on github - https://github.com/or1426/Clifford-T-estimator

References

[1] Scott Aaronson and Daniel Gottesman. "Improved simulation of stabilizer circuits". In: *Phys. Rev. A* 70 (5 Nov. 2004), p. 052328. DOI: 10.1103/PhysRevA.70.052328. URL: https://link.aps.org/doi/10.1103/PhysRevA.70.052328.

[2] Héctor Abraham et al. *Qiskit: An Open-source Framework for Quantum Computing.* 2019. DOI: 10.5281/zenodo.2562110.

[3] Sergey Bravyi et al. "Simulation of quantum circuits by low-rank stabilizer decompositions". In: *Quantum* 3 (2019), p. 181. DOI: 10.22331/q-2019-09-02-181.

[4] M. Hebenstreit et al. "All pure fermionic non-Gaussian states are magic states for matchgate computations". In: *Phys. Rev. Lett* 123 (2019).

[5] Barbara M. Terhal and David P. DiVincenzo. "Classical simulation of noninteracting-fermion quantum circuits". In: *Phys. Rev. A* 65 (2002).