Fluctuation-dissipation relations for thermodynamic distillation processes

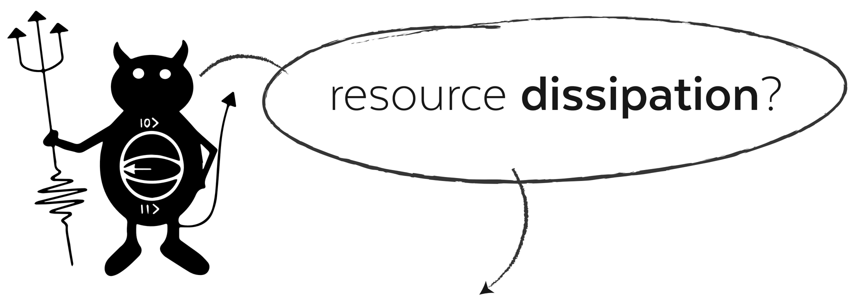
Tanmoy Biswas[1], A. de Oliveira Junior[2], Michał Horodecki[1], Kamil Korzekwa[2]

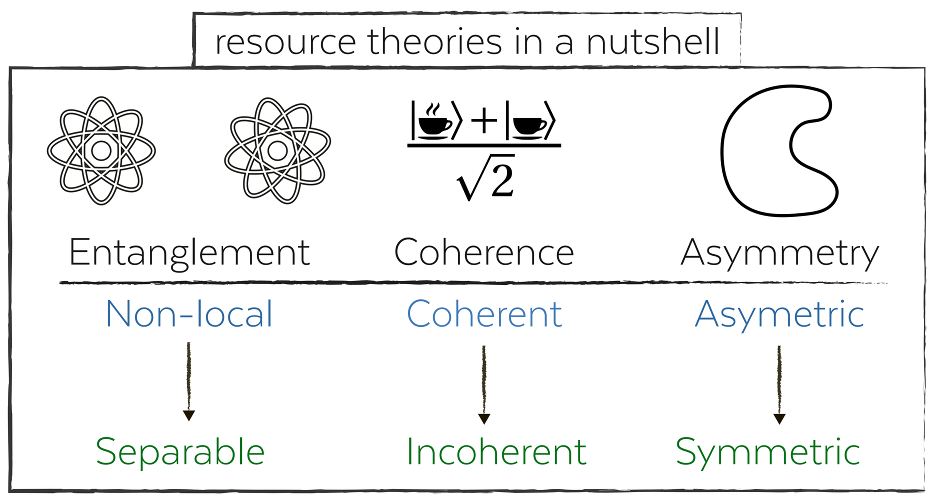
[1]International Centre for Theory of Quantum Technologies, University of Gdansk, Wita Stwosza 63, 80-308 Gdansk, Poland
[2]Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, 30-348 Krakow, Poland

Background

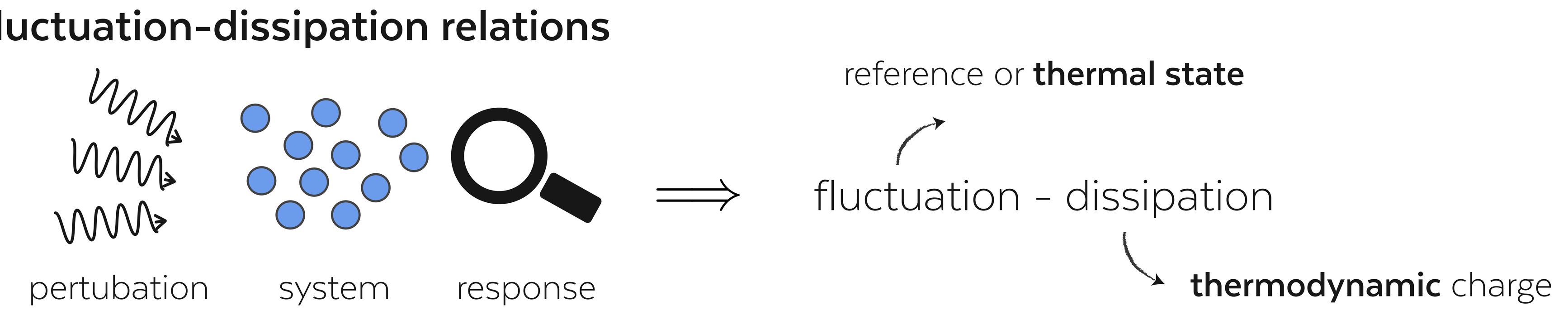
Goal: study free energy dissipation and characterise optimal thermodynamic state transformation protocols.

Framework: resource-theoretic ordering of quantum states.



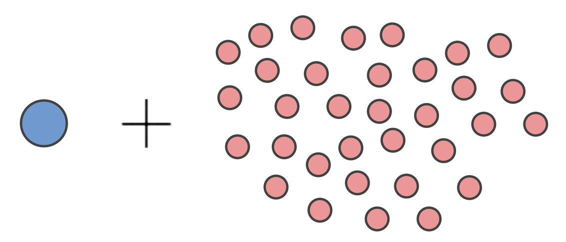


Fluctuation-dissipation relations



Resource theory of thermodynamics

Identifying the set of **thermodynamically-free states**



$$\Longrightarrow \quad \gamma = \frac{e^{-\beta H}}{Z}, \quad Z = \mathrm{Tr}(e^{-\beta H})$$

(ρ, H) $\quad (\gamma_E, H_E)$

Thermodynamic transformations are modelled by **thermal operations**

$$\mathcal{E}(\rho) = \mathrm{Tr}_E(U(\rho\otimes\gamma_E)U^\dagger) \quad \text{with} \quad [U, H\otimes \mathbb{1}_E + \mathbb{1}\otimes H_E] = 0$$

Relevant **information-theoretic** quantities

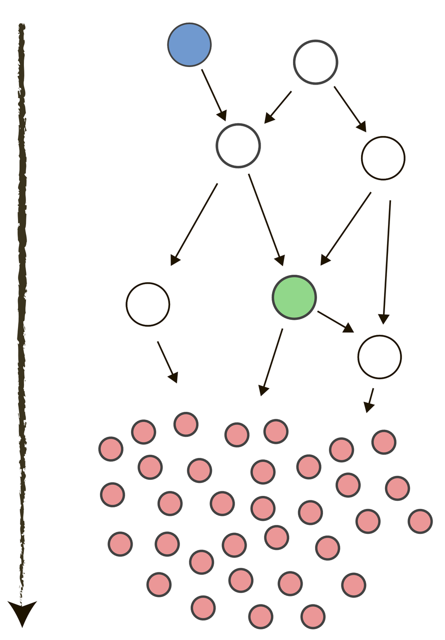
i. $D(\rho\|\gamma) = \mathrm{Tr}(\rho(\log\rho - \log\gamma))$ Generalised **free energy** (monotone)

ii. $V(\rho\|\gamma) = \mathrm{Tr}\left(\rho\left(\log\rho - \log\gamma - D(\rho\|\gamma)\right)^2\right)$ **Free energy fluctuations**

Thermodynamic distillation process

For initial state ρ, target state σ, inverse temperature $\beta \Longrightarrow \mathcal{E}(\rho) = \sigma$.

out of equilibrium



equilibrium

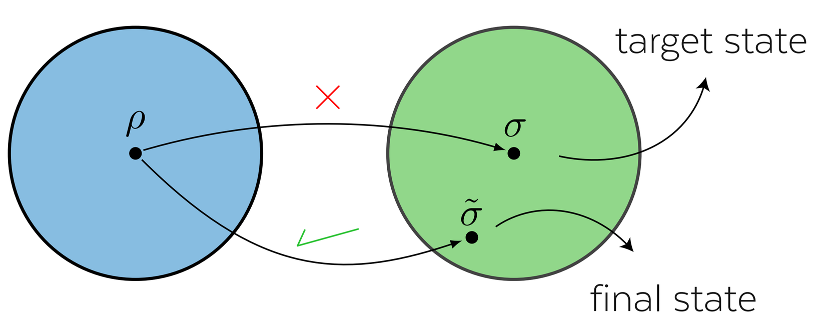
! General answer not known beyond the simplest qubit case.
Phys. Rev. X 5, 021001 (2015)
Nat.Commun. 6, 7689 (2015)

! For **energy-incoherent** states.
Nat.Commun. 4, 2059 (2013)

$$\mathcal{E}(\rho) = \sigma : \mathbf{p} \succ^\beta \mathbf{q}$$

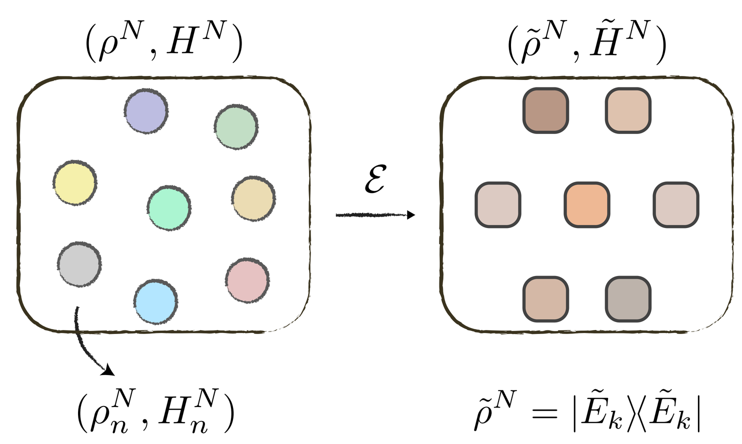
ϵ -approximate **interconversion** problem:

! Approximate interconversion problem with **finite** system.
Quantum, vol. 2, p.108, (2018)



? For **general** states.
? **Dissipation** of resources.
? **Different** subsystems.
? **Optimal** processes.

ϵ -approximate **thermodynamic distillation process**:



Relevant **thermo information-theoretic** quantities

iii. $\Delta F^N := \frac{1}{\beta}\left(\sum_{n=1}^N D(\rho_n^N\|\gamma_n^N) - D(\tilde\rho^N\|\tilde\gamma^N)\right)$ Free energy **difference**

iv. $\sigma^2(F^N) := \frac{1}{\beta^2}\sum_{n=1}^N V(\rho_n^N\|\gamma_n^N)$ Free energy **fluctuations**

Results

Theorem. For a distillation setting with energy incoherent or pure initial states, the transformation error of the approximate distillation process in the asymptotic limit is given by

$$\lim_{N\to\infty}\epsilon_N = \lim_{N\to\infty}\Phi\left(-\frac{\Delta F^N}{\sigma(F^N)}\right).$$

Moreover, for any N there exists an approximate distillation process with an enegy incoherent initial state and the transformation error bounded by

$$\epsilon_N \le \Phi\left(-\frac{\Delta F^N}{\sigma(F^N)}\right) + \frac{C\kappa^3(F^N)}{\sigma^3(F^N)}.$$

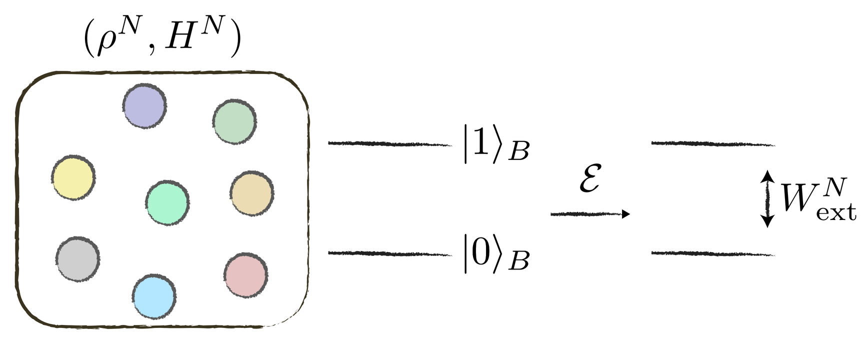
The amout of free energy dissipated in the above setting satisfies

$$F_{\mathrm{diss}}^{\mathrm{tot}} = a(\epsilon)\,\sigma^{\mathrm{tot}}(F) \quad \text{with } a(\epsilon) = -\Phi^{-1}(\epsilon)(1-\epsilon) + \frac{\exp\left(\frac{-(\Phi^{-1}(\epsilon))^2}{2}\right)}{\sqrt{2\pi}},$$

and Φ^{-1} being the inverse of the Gaussian cumulative distribution

Aplications

■ **Optimal work extraction**



Applying the above *Theorem*, the extracted work for a given error is given by:

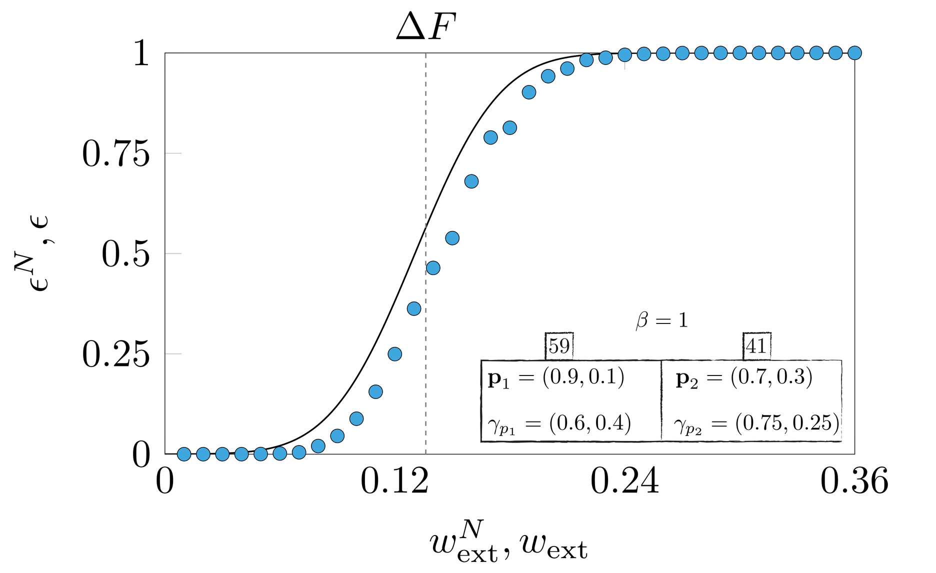
$$W_{\mathrm{ext}}^N \simeq F^N + \sigma(F^N)\Phi^{-1}(\epsilon).$$

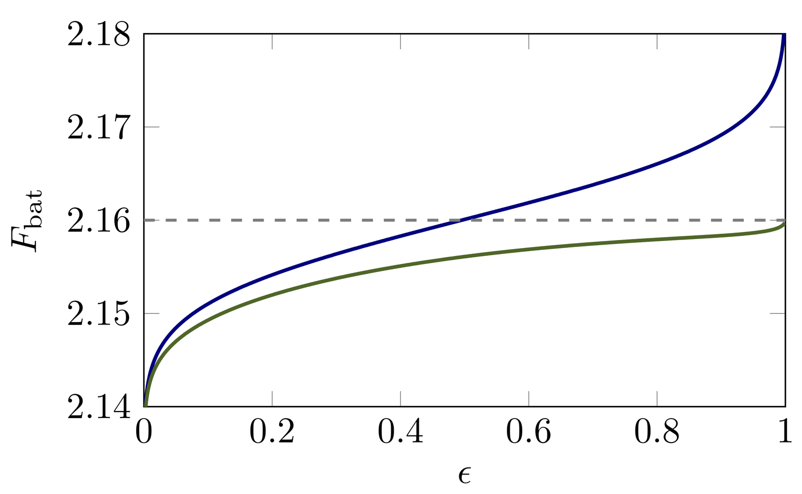
energy-incoherent states

The optimal amount of work extracted from N pure quantum systems up to second-order is given by:

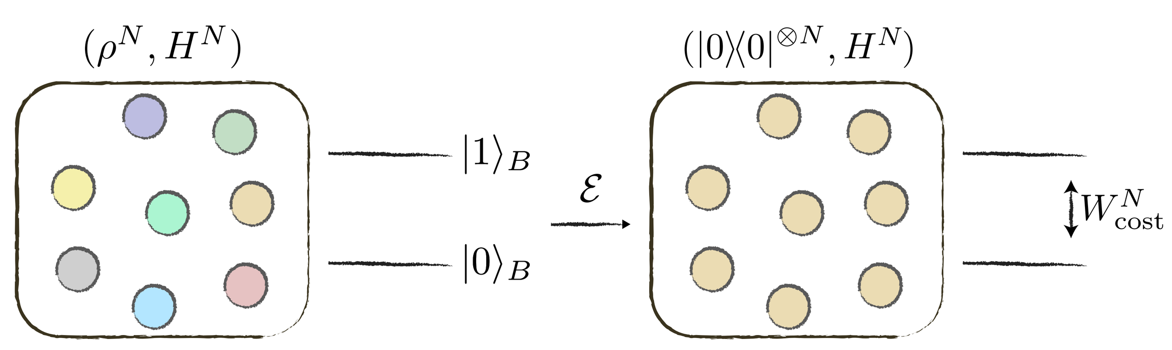
$$W_{\mathrm{ext}} \simeq N\left(\langle H\rangle_\psi + \frac{\log Z}{\beta} + \frac{\langle H^2\rangle_\psi - \langle H\rangle_\psi^2}{\sqrt{N}}\Phi^{-1}(\epsilon)\right).$$

pure states





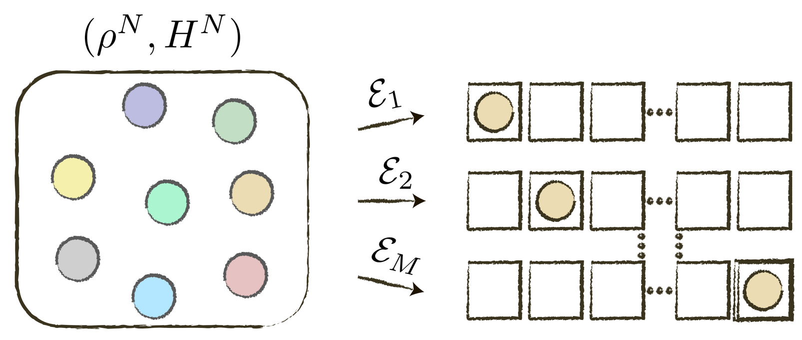
■ **Optimal information erasure**



The above *Theorem* also provides the second-order asymptotics for the cost of erasure:

$$W_{\mathrm{cost}}^N \simeq \frac{S(\rho^N)}{\beta} - \sigma(F^N)\Phi^{-1}(\epsilon).$$

■ **Optimal thermodynamically-free communication rate**



The optimal number of messages that can be encoded into in a thermodynamically-free way:

! This result is valid for either pure or incoherent states!

$$R(\rho^{\otimes N}, \epsilon) \simeq D(\rho\|\gamma) + \frac{\sqrt{V(\rho\|\gamma)}}{\sqrt{N}}\Phi^{-1}(\epsilon).$$

We acknowledge financial support by the Foundation for Polish Science through TEAM-NET project (contract no. POIR.04.04.00- 00-17C1/18-00).

FNP Foundation for Polish Science