Coherent thermodynamics beyond first-order asymptotics

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Outline

I. Statement of the problem

II. Motivation

III. How we solve it

IV. Results

V. Summary

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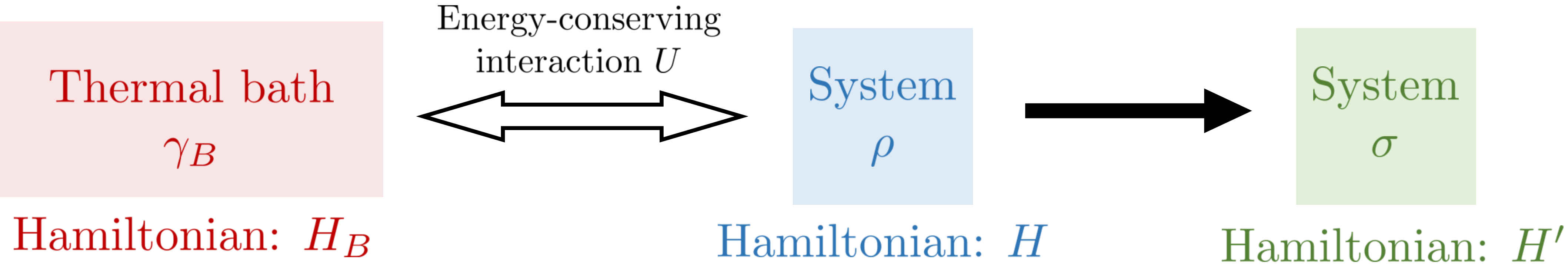
Statement of the problem

Thermodynamic transformations modelled by **thermal operations***:

$$\mathcal{E}^{T}(\cdot) = \mathrm{Tr}_{B'}\left(U\left(\cdot \otimes \gamma_B\right)U^{\dagger}\right) \quad \text{with} \quad [U, H + H_B] = 0$$



Gibbs state γ of the system at temperature T: $\quad \gamma = e^{-\frac{H}{T}}/\mathcal{Z}, \quad \mathcal{Z} = \mathrm{Tr}\left(e^{-\frac{H}{T}}\right)$

Note: all results with units such that $k_B = 1$.

*M. Horodecki, J. Oppenheim Nature Commun. **4**, 2059 (2013)

Statement of the problem

State interconversion: Initial state ρ, target state σ, background temperature T

Single-shot interconversion: Does there exist $\mathcal{E}^T$ such that $\mathcal{E}^T(\rho) = \sigma$?

Many-copies interconversion: Does there exist $\mathcal{E}^T$ such that $\mathcal{E}^T\left(\rho^{\otimes n}\right) \approx_\epsilon \sigma^{\otimes R_n n}$?

(large but finite n)

Optimal rate R_n for error ϵ?

Incoherent interconversion: $[\rho, H] = [\sigma, H'] = 0$

(states represented by: $\boldsymbol{p} = \mathrm{eig}(\rho)$, $\boldsymbol{q} = \mathrm{eig}(\sigma)$)

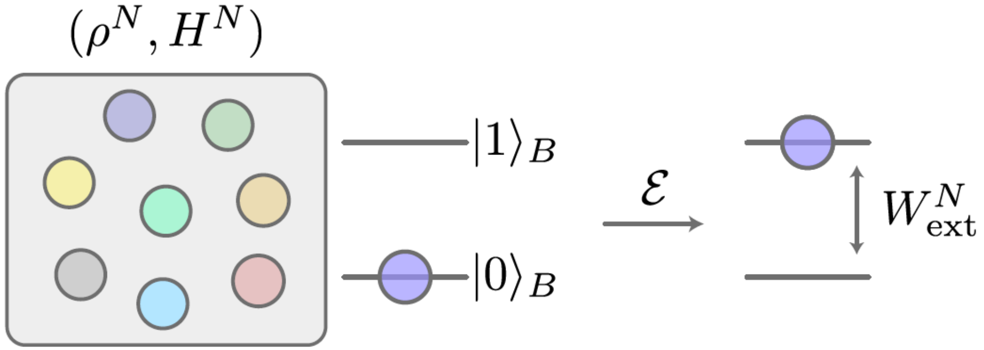
$[\gamma, H] = 0$

(thermal state represented by: $\boldsymbol{\gamma} = \mathrm{eig}(\gamma)$)

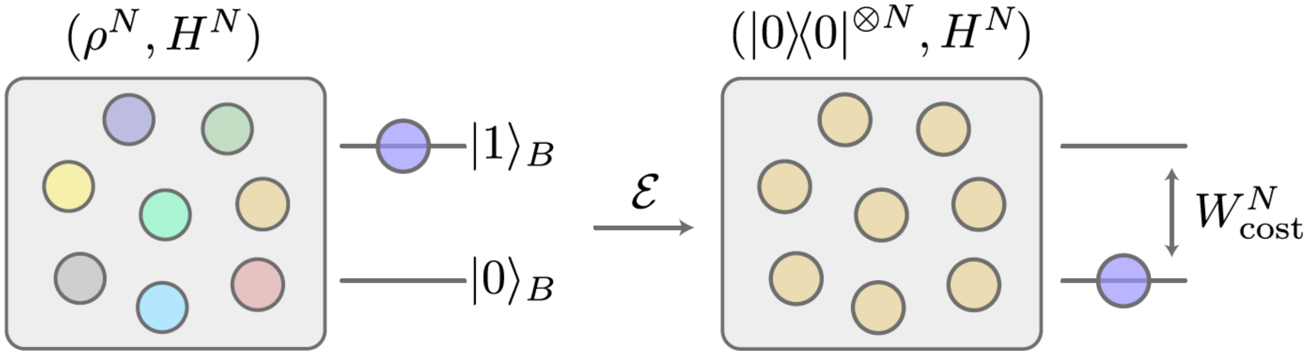
Motivation

Thermodynamic protocols are various instances of state interconversion problem

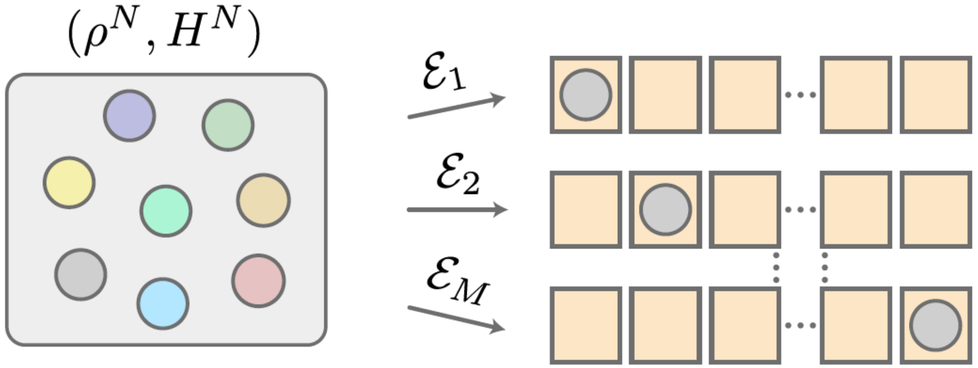
Work extraction



Information erasure

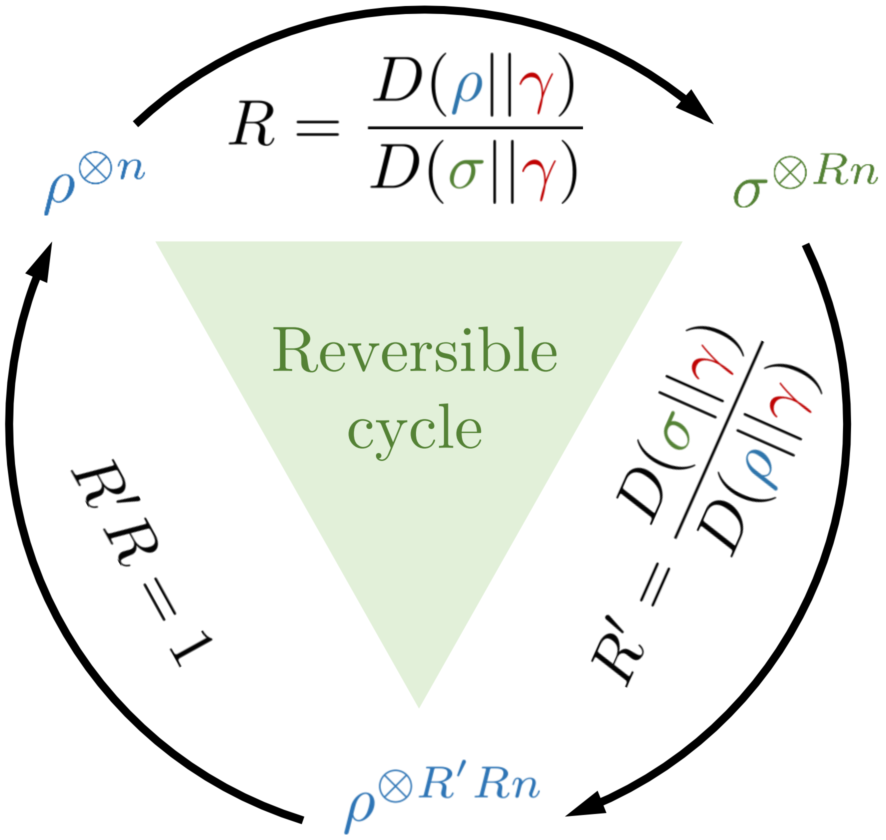


Thermodynamically-free communication



Motivation

Asymptotic rate for $n \to \infty$*: $$R_\infty(\rho \to \sigma) = \frac{D(\rho\|\gamma)}{D(\sigma\|\gamma)}$$



Relative entropy: $$D(\rho\|\gamma) := \mathrm{Tr}\left(\rho(\log\rho - \log\gamma)\right)$$

Physical interpretation: $$\frac{1}{T}\Big[\underbrace{\langle E\rangle_\rho - TS(\rho)}_{\substack{\text{Free energy}\\ F = U - TS}} - \underbrace{(-T\log\mathcal{Z})}_{\text{Free energy of }\gamma}\Big]$$

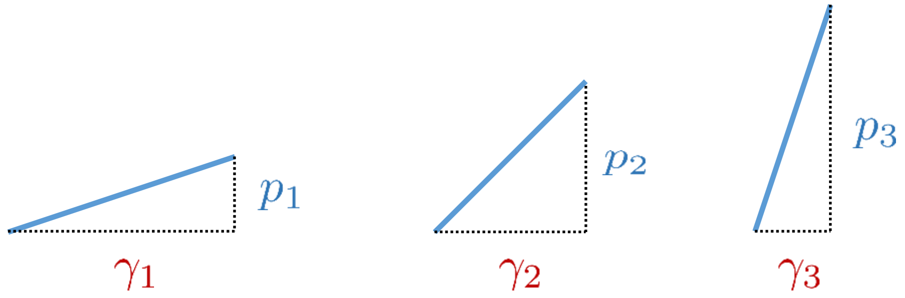
First-order asymptotics (thermodynamic limit):
No dissipation of free energy!

*F. Brandão *et al.*,
Phys. Rev. Lett. 111, 250404 (2013)

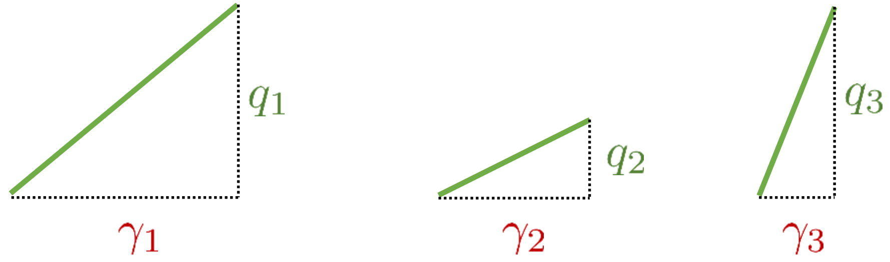
How we solve it

Incoherent interconversion completely described by **thermomajorisation***:

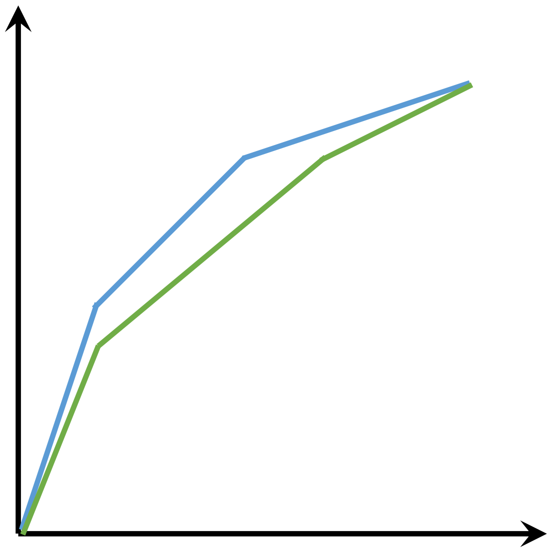
Lorenz curve segments for the initial state $\boldsymbol{p}$:



Lorenz curve segments for the target state $\boldsymbol{q}$:



Form convex Lorenz curves



Interconversion possible iff the initial curve is always above the target curve

*M. Horodecki, J. Oppenheim Nature Commun. **4**, 2059 (2013)

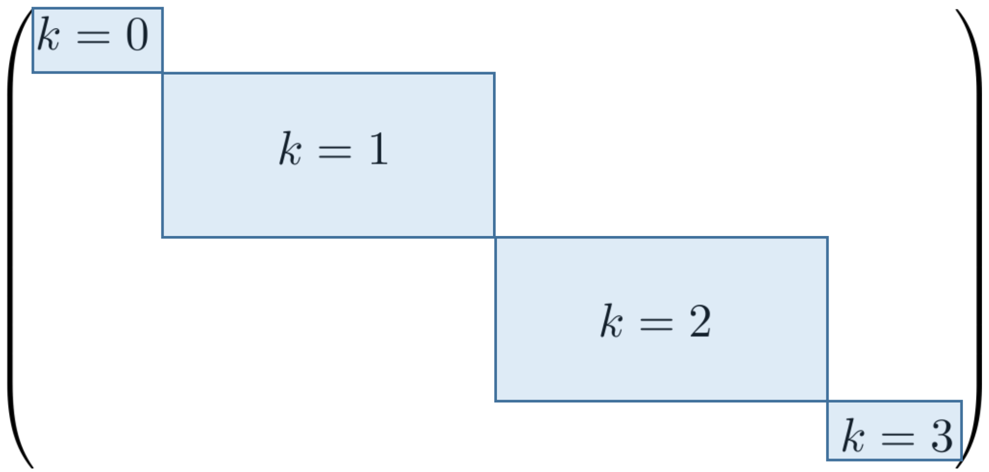
How we solve it

Consider a coherent qubit state: $\rho = \begin{pmatrix} p & c \\ c^* & 1-p \end{pmatrix}$

Then, dephasing many copies means:

Incoherent state

$\rho^{\otimes 3} =$



Each block can be diagonalised with energy-preserving unitaries

$$\begin{pmatrix} \lambda_0^1 & & & & & & & \\ & \lambda_1^1 & & & & & & \\ & & \lambda_1^2 & & & & & \\ & & & \lambda_1^3 & & & & \\ & & & & \lambda_2^1 & & & \\ & & & & & \lambda_2^2 & & \\ & & & & & & \lambda_2^3 & \\ & & & & & & & \lambda_3^1 \end{pmatrix} =: \boldsymbol{\lambda}$$

As $n \to \infty$ such dephasing pre-processing "kills" only $O(\log n)$ of free energy!

(proof using hypothesis testing approach to the interconversion problem)

Results

Optimal conversion rate R_n with constant error ϵ:

$$R_n(\epsilon) = R_\infty + \sqrt{\frac{V(\rho\|\gamma)}{D(\sigma\|\gamma')^2}}\,\frac{S_\nu^{-1}(\epsilon)}{\sqrt{n}} + o\left(\frac{1}{\sqrt{n}}\right)$$

Reversibility parameter:

$$\nu = \frac{V(\sigma\|\gamma)/D(\sigma\|\gamma)}{V(\rho\|\gamma)/D(\rho\|\gamma)}$$

Relative entropy variance: $$V(\rho\|\gamma) := \mathrm{Tr}\left(\rho\,(\log\rho - \log\gamma)^2\right) - D(\rho\|\gamma)^2$$

Inverse of *sesquinormal* distribution: $$S_\nu^{-1}(\epsilon) = \inf_{x\in(\epsilon,1)} \sqrt{\nu}\,\Phi^{-1}(x) - \Phi^{-1}(x-\epsilon)$$

Limiting cases: $$S_0^{-1}(\epsilon) = \lim_{\nu\to\infty}\frac{1}{\sqrt{\nu}}S_\nu^{-1}(\epsilon) = \Phi^{-1}(\epsilon) \qquad S_1^{-1}(\epsilon) = 2\Phi^{-1}\left(\frac{1+\epsilon}{2}\right)$$

Results

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Optimal performance of thermodynamic protocols employing interference effects:

Extractable work:

$$w \simeq \frac{1}{\beta}\left(D(\rho\|\gamma) + \sqrt{\frac{V(\rho\|\gamma)}{n}}\Phi^{-1}(\epsilon)\right)$$

Work cost of information erasure:

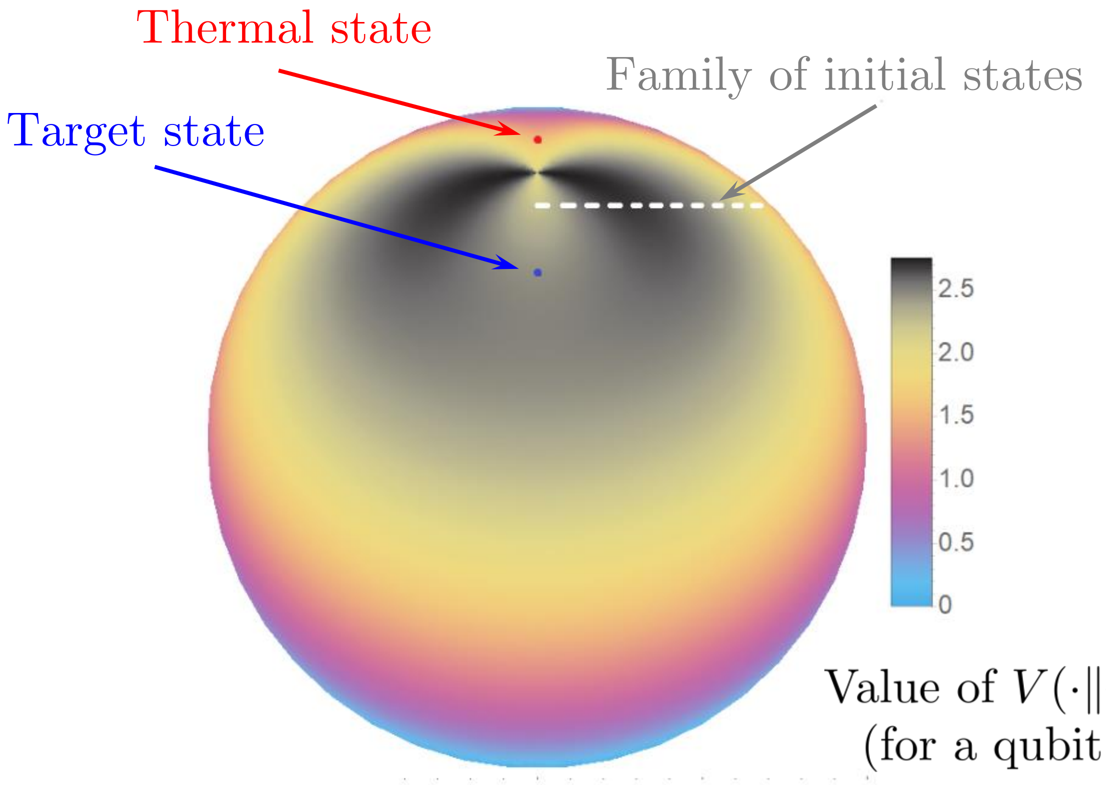
$$w_{\text{cost}} \simeq \frac{1}{\beta}\left(S(\rho) - \sqrt{\frac{V(\rho)}{n}}\Phi^{-1}(\epsilon)\right)$$

Number of bits that can be communicated without a thermodynamic cost:

$$\frac{\log M(\rho^{\otimes n}, \epsilon)}{n} \simeq D(\rho\|\gamma) + \sqrt{\frac{V(\rho\|\gamma)}{n}}\Phi^{-1}(\epsilon),$$

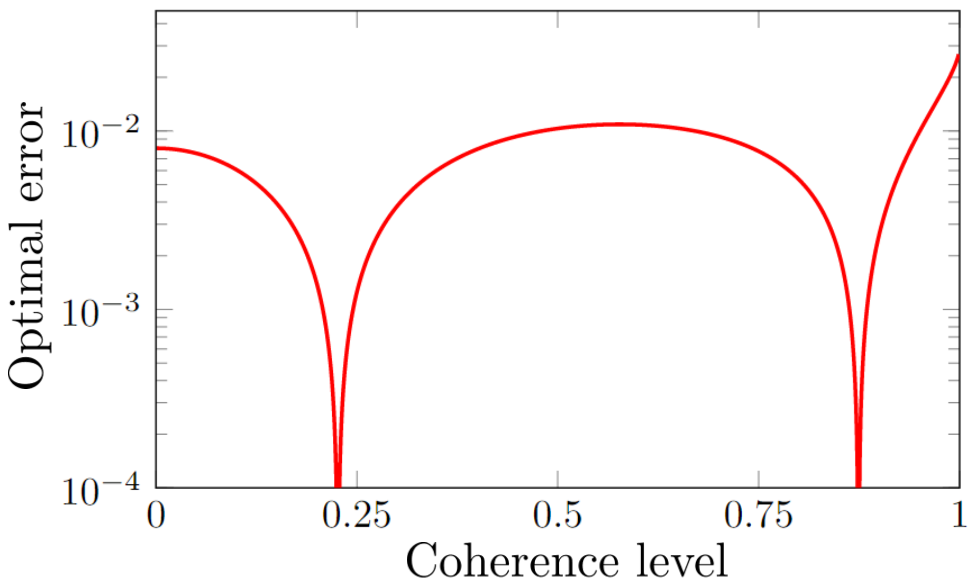
Results

Predicting coherent resonance phenomenon:



Value of $V(\cdot\|\gamma)/D(\cdot\|\gamma)$ (for a qubit system)

Transformation with the asymptotic rate



Recall reversibility parameter:

$$\nu = \frac{V(\sigma\|\gamma)/D(\sigma\|\gamma)}{V(\rho\|\gamma)/D(\rho\|\gamma)}$$

Summary

- Asymptotic analysis for transformation of quantum dichotomies
- Second-order analysis in all error regimes for trace distance
- Tight for general non-commuting inputs in all-but-one regime
- Opens door to study role of coherence in resource theories like thermodynamics
- New (much faster!) derivation for entanglement transformations

Thank you!