Optimizing thermalizations

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TEAM-NET

Outline

1. Motivation
2. Statement of the problem
3. Main technical tool
4. Results
5. Applications
6. Outlook

Based on:

arXiv:2111.12130 – mathematical framework

arXiv:2202.12616 – applications

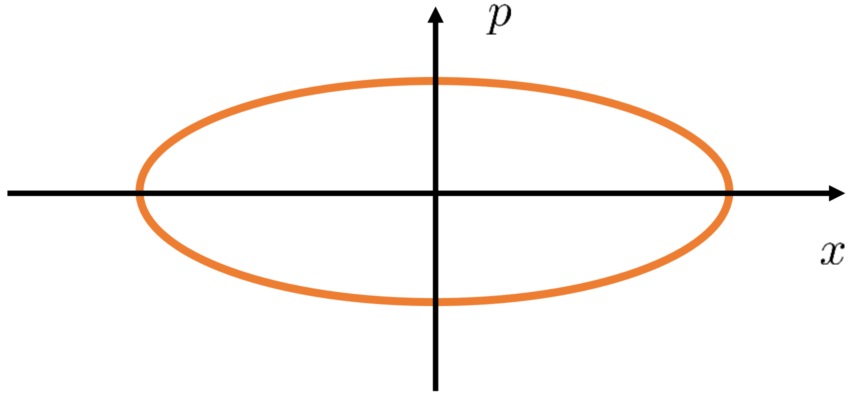
In collaboration with:

Matteo Lostaglio
PsiQuantum

Motivation

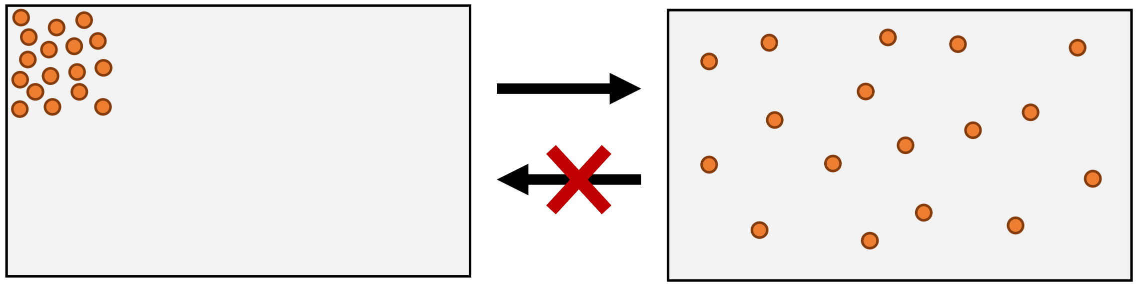
What can we say about the dynamics without solving equations of motion?

Closed systems



Energy conservation

Open systems

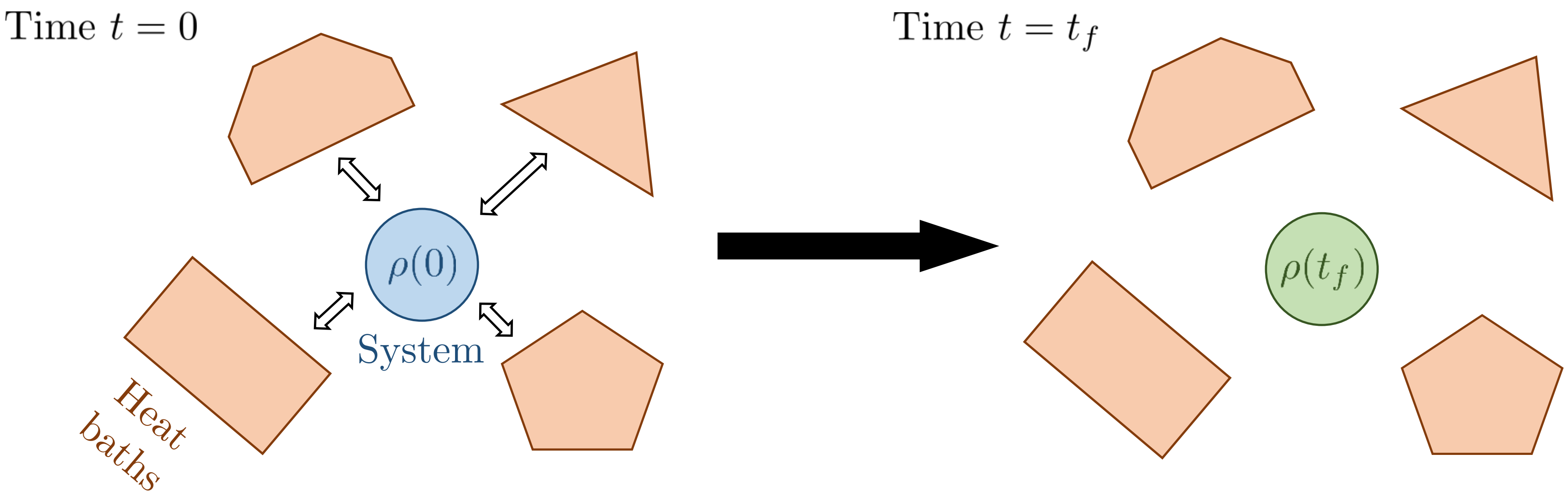


Entropy growth

Quantum thermodynamics:

Using minimal assumptions of the quantum theory, find constraints on the evolution of a quantum system interacting with thermal baths

Statement of the problem



Original question: Given $\rho(0)$ and $\Leftrightarrow$ denoting arbitrary energy-conserving unitary, what can $\rho(t_f)$ be?

Thermal operations

M. Horodecki, J. Oppenheim
Nature Commun. **4**, 2059 (2013)

Our question: Given $\rho(0)$ and $\Leftrightarrow$ denoting Markovian energy-conserving interaction, what can $\rho(t_f)$ be?

Markovian thermal processes

Formal statement of the problem

General Markovian open quantum dynamics:

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \mathcal{L}_t(\rho(t)), \qquad \text{where} \quad H = \sum_{i=1}^{d} E_i \, |E_i\rangle\langle E_i|$$

$$\text{and} \qquad \mathcal{L}_t(\rho) = \sum_{i=1}^{d^2-1} r_i(t) \left(L_i(t)\rho L_i^\dagger(t) - \frac{1}{2}\{L_i^\dagger(t) L_i(t), \rho\} \right)$$

Markovian thermal process (MTP) defined by additional two properties:

- Stationary thermal state: $\forall t: \quad \mathcal{L}_t \gamma = 0, \quad \text{where} \quad \gamma = e^{-\beta H}/\mathrm{Tr}\left(e^{-\beta H}\right)$
- Covariance: $\forall t, \rho: \quad \mathcal{L}_t([H,\rho]) = [H, \mathcal{L}_t(\rho)]$

Microscopic derivations of quantum master equations usually lead to MTPs

Solving for possible final energy populations:

$$\boldsymbol{p}(0) \overset{\text{MTP}}{\longmapsto} \boldsymbol{p}(t_f), \quad \text{where} \quad p_i(t) = \langle E_i|\rho(t)|E_i\rangle$$

Main technical tool

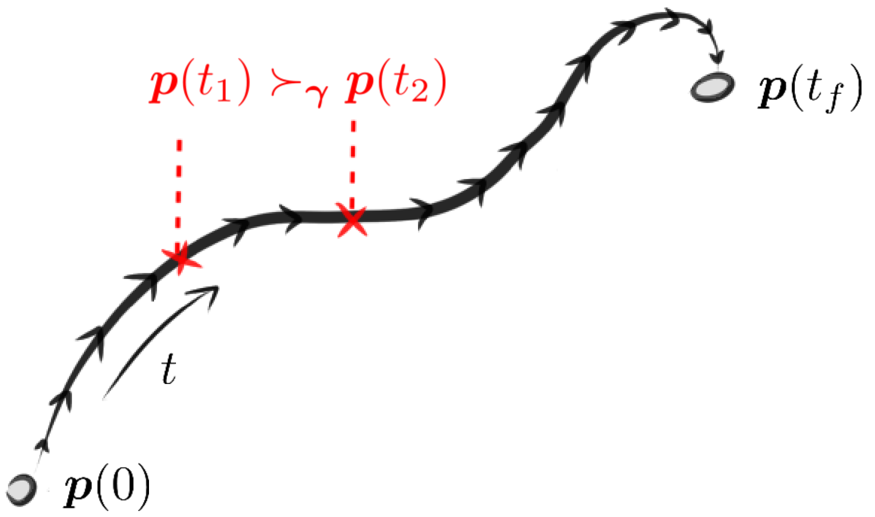
Continuous thermomajorisation:

$$\boldsymbol{p}(0) \gg_{\gamma} \boldsymbol{p}(t_f)$$

iff there exists a thermomajorising trajectory $\boldsymbol{p}(t)$ such that:

$$\forall t_1, t_2 \in [0, t_f]: \quad t_1 \leq t_2 \Rightarrow \boldsymbol{p}(t_1) \succ_{\gamma} \boldsymbol{p}(t_2)$$

where $\succ_{\gamma}$ denotes thermomajorisation ordering (majorisation relative to $\boldsymbol{\gamma}$)



It yields a complete description of population dynamics:

$$\boldsymbol{p}(0) \xrightarrow{\text{MTP}} \boldsymbol{p}(t_f) \quad \text{if and only if} \quad \boldsymbol{p}(0) \gg_{\gamma} \boldsymbol{p}(t_f)$$

Results

Exhaustive H-type theorem:

A dynamical evolution $\boldsymbol{p}(t)$ of populations can be generated by a Markovian thermal process if and only if:

$$\forall a \in [0,1]: \quad \frac{d\Sigma_a(t)}{dt} \geq 0, \qquad \text{where} \qquad \Sigma_a := -\sum_{i=1}^{d} \left| p_i(t) - a\frac{\gamma_i}{\gamma_d} \right|.$$

Universality of elementary thermalisations:

$\boldsymbol{p}(0) \xmapsto{\text{MTP}} \boldsymbol{p}(t_f)$ is possible if and only if there exists a sequence of elementary thermalisations such that:

$$\boldsymbol{p}(t_f) = T^{i_f, j_f}(\lambda_f) \ldots T^{i_1, j_1}(\lambda_1)\boldsymbol{p}(0), \quad \text{where} \quad T^{i,j}(\lambda) = \begin{bmatrix} (1-\lambda) + \frac{\lambda\gamma_i}{\gamma_i+\gamma_j} & \lambda\frac{\gamma_i}{\gamma_i+\gamma_j} \\ \lambda\frac{\gamma_j}{\gamma_i+\gamma_j} & (1-\lambda) + \frac{\lambda\gamma_i}{\gamma_i+\gamma_j} \end{bmatrix} \oplus \mathbf{1}_{\setminus(i,j)}$$

Results

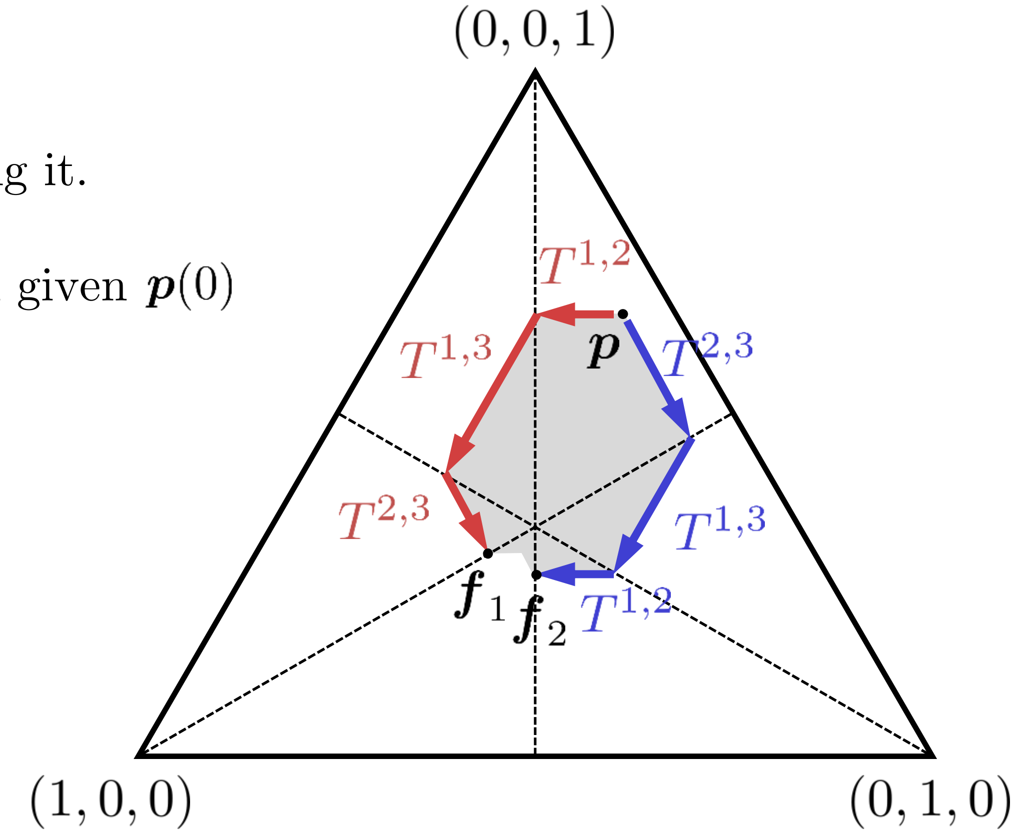
Algorithmic verification of $\boldsymbol{p}(0) \overset{\mathrm{MTP}}{\longmapsto} \boldsymbol{p}(t_f)$:

- Only finite set of conditions needs to be verified.
- If the path exists, the algorithm returns the Lindbladian realising it.
- One can also find the full set of states achievable via MTP from given $\boldsymbol{p}(0)$

github.com/KorzekwaKamil/continuous_thermomajorisation

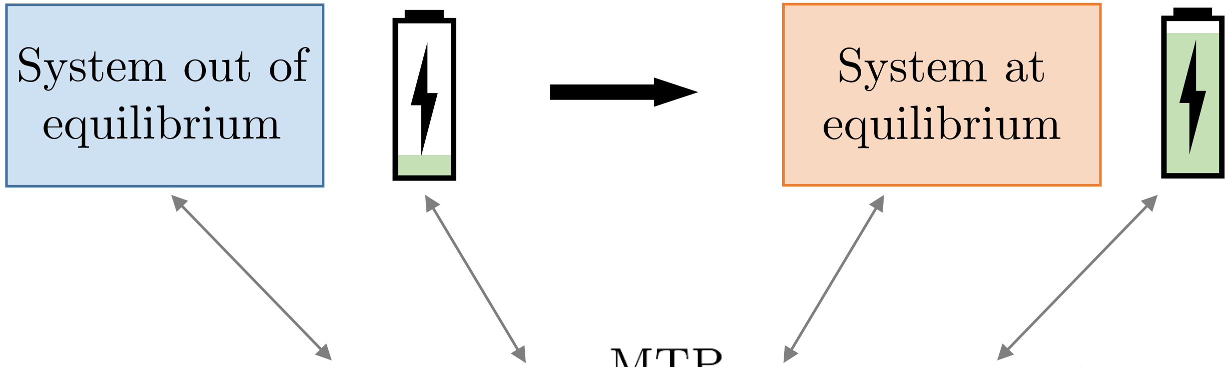
Idea 1: when initial and final state have the same ordering, it's easy

Idea 2: when changing orderings, there is a unique optimal way to do it



Applications

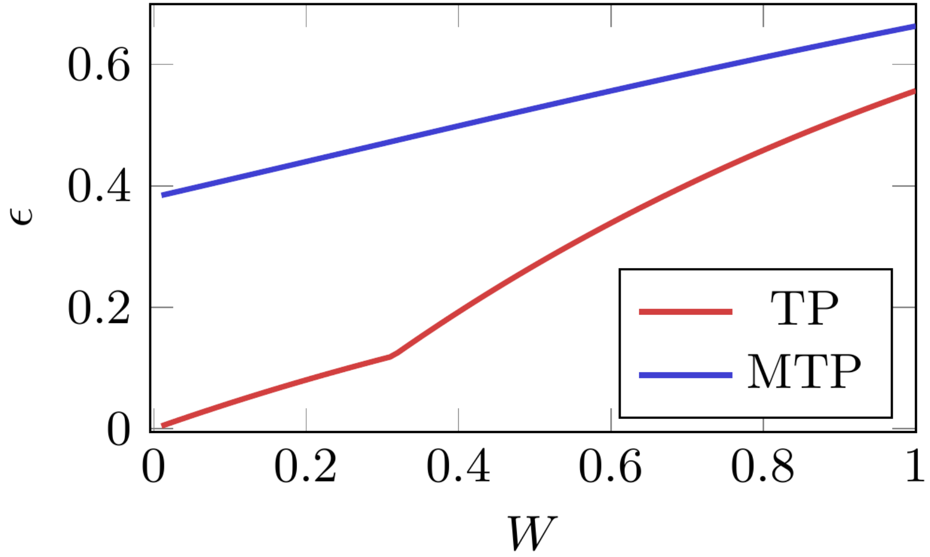
Role of memory in work extraction:



$$\boldsymbol{p} \otimes (1,0) \overset{\text{MTP}}{\longmapsto} \boldsymbol{\gamma} \otimes (\epsilon, 1-\epsilon)$$

versus

$$\boldsymbol{p} \otimes (1,0) \overset{\text{TP}}{\longmapsto} \boldsymbol{\gamma} \otimes (\epsilon, 1-\epsilon)$$



System spectrum $\{0, 1\}$
Battery spectrum $\{0, W\}$
System initially thermal with $\beta_S = 2$
Bath with $\beta_E = 1$

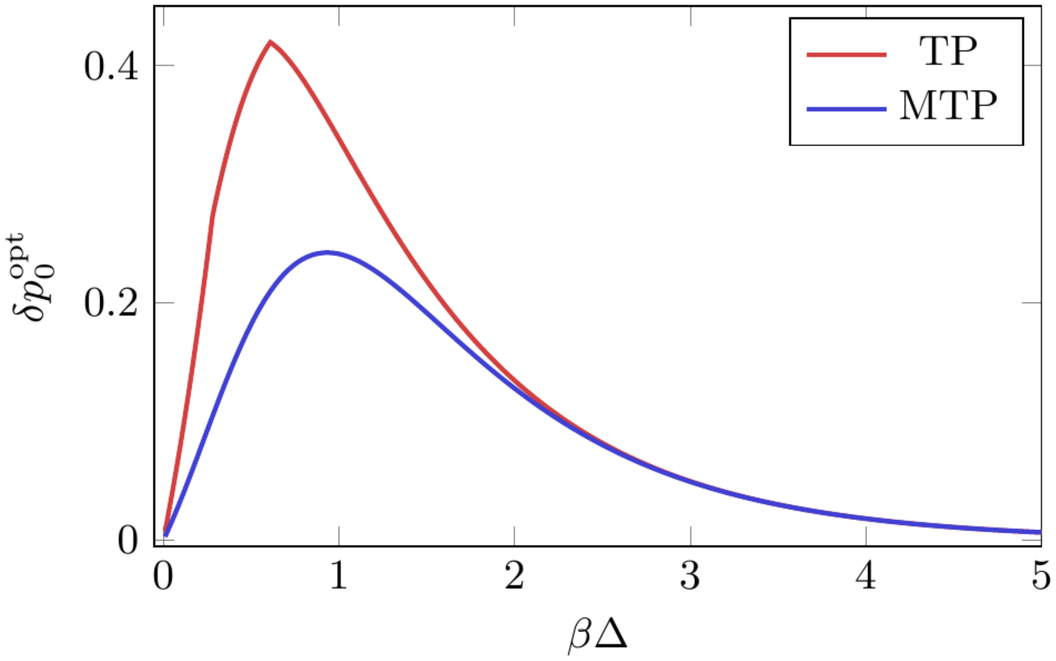
Applications

Role of memory in cooling:

One step of heat-bath algorithmic cooling protocol:

- Take a thermal system and unitarily invert its populations.
- Interact it with the bath and try to maximise ground state population.

Again, we can compare optimal MTP with optimal TP protocols.



System spectrum $\{0, \Delta, 2\Delta, 3\Delta\}$
System initially in equilibrium with bath at β

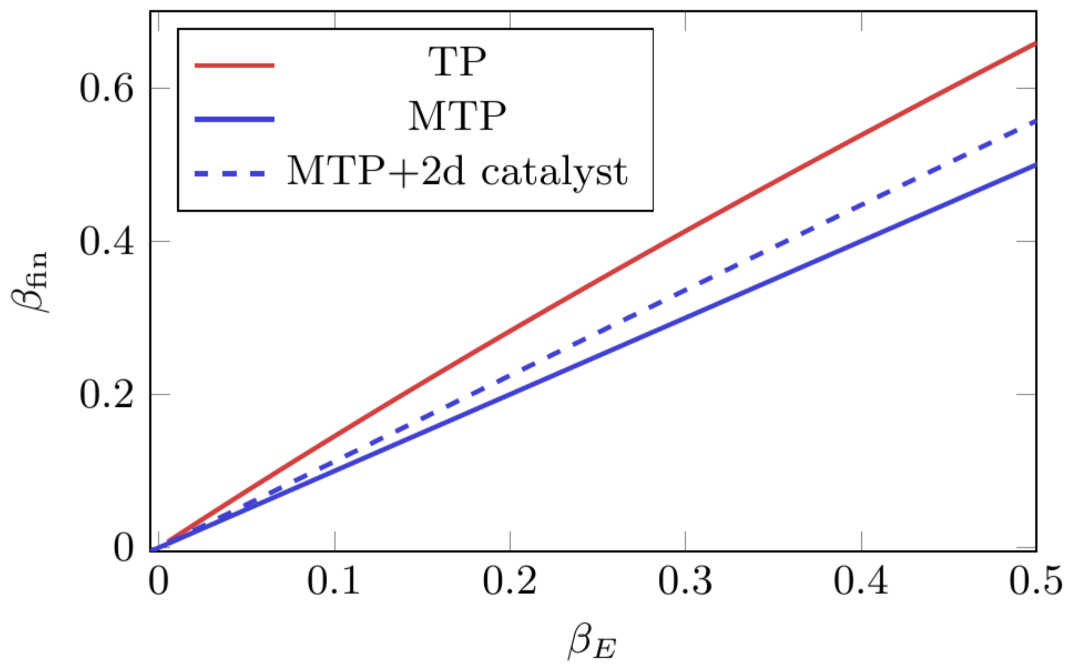
Applications

Catalysts and memory in thermodynamic protocols

Catalyst $\boldsymbol{c}$ is a system that is returned unchanged at the end of the process:

$$\boldsymbol{p} \otimes \boldsymbol{c} \xmapsto{\text{MTP}} \boldsymbol{q} \otimes \boldsymbol{c}$$

Thermal catalysts can be used as a memory that enhances or unlocks otherwise impossible tasks to be performed, with catalyst's dimension quantifying the amount of memory.



System and catalyst spectrum $\{0, 1\}$
System initially thermal with $\beta_S = \beta_E/2$
Bath and catalyst thermal with β_E

Outlook

1. Apply to study non-Markovian boosts to relevant processes (see: arXiv:2103.14534)
2. Understand the asymptotic behaviour of continuous thermomajorisation.
3. Extend the formalism to treat states with coherence.
4. Optimise the runtime of the algorithmic verification procedure.

See more:

Continuous thermomajorisation and a complete set of laws for Markovian thermal processes
M. Lostaglio, K. Korzekwa
arXiv:2111.12130 (2021)

Optimizing thermalizations
K. Korzekwa, M. Lostaglio
arXiv:2202.12616 (2022)

Thank you!